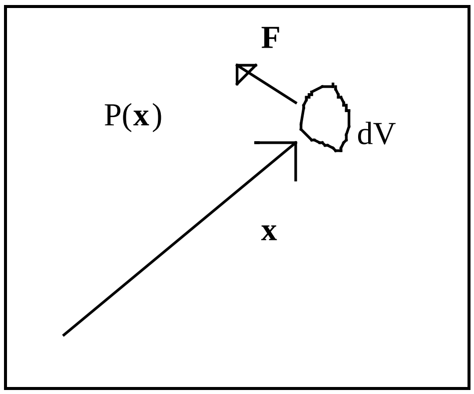
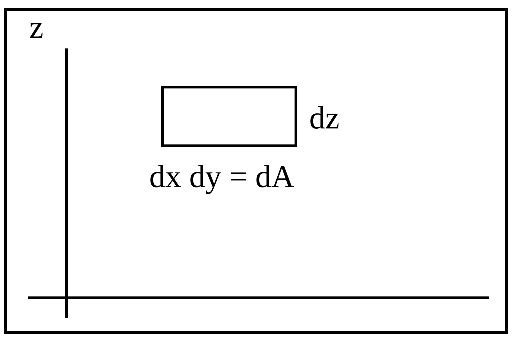
Force due to pressure

The net force $\mathbf{F}$ on an element of fluid of volume dV due to the rest of the fluid with pressure field $P(\mathbf{x})$ is $\mathbf{F} = -dV\,\nabla P$.



To see this, look at the z direction first.



$$F_z = P(z)\,dA - P(z+dz)\,dA$$
$$= -dA\,dz\frac{\partial P}{\partial z} = -dV\frac{\partial P}{\partial z} \qquad (dz \to 0)$$

With similar results for x and y, we assemble the components to

$$\mathbf{F} = -dV\left(\hat{\mathbf{x}}\frac{\partial P}{\partial x} + \hat{\mathbf{y}}\frac{\partial P}{\partial y} + \hat{\mathbf{z}}\frac{\partial P}{\partial z}\right) = -dV\nabla P$$

If there are no other forces acting on the fluid, and it is static, then we must have $\nabla P = 0$ i.e. constant pressure.

Gravity

Frequently, the graviational force on the fluid must be included. Recall that $\mathbf{g}$ is the vector acceleration due to gravity. The magnitude is the magnitude of the acceeration, and $\mathbf{g}$ points in the direction of the gravitational force. This can be related to the gravitational potential ϕ via $\mathbf{g} = -\nabla\phi$. The force due to gravity is $\mathbf{F}_g = m\mathbf{g} = \rho dV\mathbf{g}$. For a static fluid, the total force on each element must be zero so $0 = \mathbf{F} + \mathbf{F}_g = dV(-\nabla P + \rho\mathbf{g})$ or $-\nabla P = -\rho\mathbf{g} = \rho\nabla\phi$.