# Viscous force

Physics 9HB, Joe Kiskis

You should read the section in the text on viscosity before you go on with this.

There you will see the relationship between the force across two planes and the coefficient of viscosity. The planes have area $A$ and are oriented perpendicular to the $y$-axis. If plane $a$ is moving in the positive $x$-direction relative to plane $b$, the force of plane $a$ on plane $b$ is in the $x$-direction

$$F_x(a\ on\ b) = -F_x(b\ on\ a) = \eta A \frac{\partial v_x}{\partial y}. \tag{1}$$

Now consider a small element of fluid with the orientations as above and with a thickness $\Delta y$ in the $y$-direction. The force on the top of the fluid element is

$$F_x(top) = \eta A \frac{\partial v_x(y + \Delta y)}{\partial y}. \tag{2}$$

On the bottom, the force *on* the element from the plane below it is

$$F_x(bottom) = -\eta A \frac{\partial v_x(y)}{\partial y}. \tag{3}$$

Thus, to first order in the small $\Delta y$, the total force due to viscosity is

$$F_x = \eta A \Delta y \frac{\partial^2 v_x}{\partial y^2}. \tag{4}$$

This shows that the viscous force to be added to the equation of motion has the form

$$\mathbf{F}_\eta = \eta dV \times (second\ \ derivatives\ \ of\ \ \mathbf{v}). \tag{5}$$

If we consider all the other directions and restrict to incompressible flow ($\nabla \cdot \mathbf{v} = 0$), the result is

$$\mathbf{F}_\eta = \eta dV \nabla^2 \mathbf{v}. \tag{6}$$

This gives another force per unit volume to be added to the right hand side of the equation of motion. The result

$$\rho[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v}] = -\nabla P + \rho \mathbf{g} + \eta \nabla^2 \mathbf{v} \tag{7}$$

is called the Navier-Stokes equation. It is the equation of motion for incompressible, viscous flow.