Quiz for lecture 4

Suppose that I have a set of states $|nlm\rangle$ and I am interested in matrix elements of the position operator $\vec{x} = (x, y, z)$. Suppose that after a fifty page calculation, I have discovered that $\langle 221|z|111\rangle = 0$. I would also like to know $\langle 220|x|11-1\rangle$. What should I do? What is the result?

Solution:
Use the Wigner-Eckart theorem!
x, y, z are the three parts of an l = 1 tensor operator.
The $l'' = 1$, $m'' = 0$ piece is z.

$$\left\langle n'l'm'\middle|O^{l''}_{m''}\middle|nlm\right\rangle = \langle O\rangle\left\langle l'm'\middle|(l''l)m''m\right\rangle$$

$$\langle 221|z|111\rangle = \langle O\rangle\langle 21|(11)01\rangle = \langle O\rangle\frac{1}{\sqrt{2}}$$

Thus $\langle O\rangle = 0$.

The operator x is a linear combination of l = 1, m = 1 and l=1, m = -1 pieces, so

$$\langle 220|x|11-1\rangle = \langle O\rangle \times (\text{linear combination of Clebsches})$$

with the *same* reduced matrix element. Thus it is also zero.