Physics 223B, winter 2014

Reading 12, for Thursday, Feb. 20

Georgi, Chs. 6 and 7

The rest of the course will be a more general and organized study of Lie algebras. However, a little background may be helpful in providing more context for the study. For that, you may want to look at

- Georgi, Ch.2
- Classical Lie groups (on our website)
- Lie groups and Lie algebras (on our website)

Section comments on Ch. 6:

- ROOTS AND WEIGHTS: More new words; there's no end to the jargon. As he says, the idea is to generalize the J_3, $J_\pm$ SU(2) method to other simple Lie algebras. The generalization of J_3 is the *Cartan subalgebra* with m (*rank of the algebra*) commuting generators H_i, $i = 1, ..., m$. The eigenvalues of these operators are the *weights* μ_i which generalize the eigenvalue M of J_3 in SU(2).
- ROOTS: The definition of the way the algebra acts on itself in the adjoint representation is in Eq. 6.8. In the adjoint representation, where the Lie algebra acts on itself by commutation, the weights are called *roots* α_i.
- RAISING AND LOWERING OPERATORS: Clearly, this generalizes the $J_\pm$ and gets a little more technical. The results that will be used often are Eqs. 6.19, 6.20, 6.21, and 6.36.

Section comments on Ch. 7:

- SU(3): This chapter does the Chapter 6 stuff for the SU(3) example. It should make things a lot clearer. Also SU(3) is itself very important because it has many applications in particle physics. The figures on pages 100 and 101 are very famous.

Reading 13, for Tuesday, Feb. 25, and Thursday, Feb. 27

Georgi, Chs. 8 and 9

This really goes to the heart of this approach to Lie algebras. One main result shows how to construct the entire Lie algebra from the simple roots. The other main result is a determination of all the irreps. of a simple Lie algebra. This is a generalization of the method used for SU(2) in Tung around p. 104. This is a big part of what we want to know. Many of the arguments are short but tricky. In later chapters, he determines what all the possible sets of simple roots are. This is the famous classification of simple Lie algebras. Depending upon what we decide to do for the last part of the course, we may or may not cover that.

Comments on Ch. 8:

- 8.1 Positive weights: The definition of a *positive* weight is given. That gives an *ordering* and the idea of a *highest weight*.
- 8.2 Simple roots: Next comes the crucial notion of *simple root*. The simple roots are a basis (but not orthonormal) in the root vector space. All the roots can be determined from the simple roots. Always pay attention to the SU(3) example. If you have that down cold, then you are doing well.
- 8.3 Constructing the algebra: Wit all the roots known, the last step in constructing the the associated generators is getting the normalization right. I.e. what are the $N_{\alpha\beta}$ in $[E_\alpha, E_\beta] = N_{\alpha\beta} E_{\alpha+\beta}$?
- 8.4 Dynkin diagrams: Dynkin diagrams are a shorthand way to list the simple roots. We will not use them much, but they are very important in the classification theorem.
- 8.5, 8.6, 8.10 and 8.11 G_2 and C_3 examples.
- 8.7, 8.8, and 8.9: Cartan matrix, etc. These contain additional techniques for finding all the roots and generators. From a theoretical perspective, it's important to know that this can be done. It's very rare that you will have to go through the process ourself in the course of a physics calculation. Almost always, you can look up the results you need for a particular algebra. However, you will need to be familiar with the fundamentals to understand what you find and to know what questions to ask.
- 8.12 Fundamental weights: Do not be fooled by the way that the discussion of *fundamental weights* is tacked onto the end of the chapter. It is very important. It gives all the irreps of the algebra. Everything about an irrep is determined by its highest weight, and this gives all the possible highest weights. Further, every highest weight is some sum of the m *fundamental weights* which are the highest weights for the *fundamental representations*. And every irrep is found in the corresponding direct product of fundamental reps. This is just great!

In Ch. 9, all this is applied to SU(3). If you get everything here, then you will know more about SU(3) than the average particle physicist and will be in good shape.

Section comments on Ch. 9:

- 9.1 Fundamental representations of SU(3): Simple roots, fundamental weights, and fundamental reps—the bricks from which everything else is built.
- 9.2 Constructing the states: This elaborates on the general method (not just SU(3)) for finding all of the states in a irrep.
- 9.3 Weyl group: This symmetry of weight diagrams can help in filling out the states of an irrep.
- 9.4 Complex conjugation: Another useful way to find a new representation (unless the starting irrep is *real*).
- 9.5 Examples of other representations: This turns the crank to get more SU(3) irreps. The ones you will encounter often are 3, 6, 8, and 10 (and their complex conjugates (except, of course, 8 which is real)).

  Quarks carry a 3 of color and antiquarks a $\bar{3}$, but all observed particles are color singlets. The light baryons made of u, d, and s quarks are 8's and 10's of SU(3) flavor while mesons are 1' and 8's. The quarks carry the flavor 3.

Reading 14, for Tuesday, Mar. 4

Georgi, Chs. 10 and 12 (But not all sections are important. Check the notes below.)

These are the last general theory chapters for the core of the course. In these chapters, some useful tools are presented. They are mainly ways to deal with direct products and their reduction in SU(3). The part to concentrate on is Sec. 10.5. The document "Notes on the tensor method for decomposition" on our website is a supplement to this section. Some of the results (if not the methods) are something every particle physicist knows by heart: $3 \times 3 = \bar{3} + 6$, $3 \times \bar{3} = 1 + 8$, $3 \times 3 \times 3 = 1 + 8 + 8 + 10$.

Section comments on Ch. 10:

- 10.1 and 10.2: The machinery of *tensors* is introduced. This is similar to what we did in Tung. Here, it is specialized to SU(3). Also there are some new aspects with the inclusion of bra vectors and the complex conjugate of the defining, fundamental irrep. (You can find a more systematic discussion in Ch. 13 of Tung.) There's some detail on how tensors transform.

- 10.3 Irreducible representations and symmetry: The special properties of the tensors that transform as the irreps that we already know about are given.
- 10.4 Invariant tensors: The existence of the *invariant tensors* δ^i_j and ϵ_{ijk} is noted. These are heavily used in the work that follows.
- 10.5 Clebsch-Gordan decomposition: This supposedly shows how the invariant tensors can be used to carry out the *decomposition* of direct product reps. This is the main point of the work. For more on this, see the document "Notes on the tensor method for decomposition" on our website.
- 10.6 through 10.10: These are concerned with technical points of less interest. The notion of *triality* is worth knowing. It can be handy occasionally.
- 10.11 The weights of (n,m): This is a graphical method for simplifying the determination of the weights and states of an SU(3) irrep.
- 10.12 Generalization of Wigner-Eckhart: The section title pretty much says it all. The two examples he does are the ones you might actually want to know for a calculation.
- 10.13 through 10.16: Way more than you ever wanted to know about SU(2). Not recommended.

Comments on Ch. 12:

This gives the cookbook rules for an alternative method for decomposing a direct product using Young tableaux. The main thing is Sec. 12.2 wherein the rules are laid out in a short, simple, and reasonably clear manner. A proof that the rules give the correct results is not included. In fact, the proof is rarely given, and I have never seen one that is comprehensible. It's a handy technique, but, personally, I don't generally use results that I can't prove and really understand myself. It's too easy to make mistakes when you don't understand what's going on. In addition, it's not fun. But no need to consider my eccentric opinions. We can try to talk about this method anyway.

Reading 15, for Tuesday, Mar. 11

Georgi, Chs. 11, 16, 17

If you are rusty on isospin, you might want to do a quick review of that first. However, Georgi would not be at the top of my list for that. You may have another high energy book that does a better job.

Chapter 11

Here we have the famous, and now venerable, generalization of $SU(2)_{flavor}$ (*i.e.* I-spin) to $SU(3)_{flavor}$. In terms of quarks, it's u and d goes to u, d, and s.

Section comments:

- 11.1 Eightfold way: The very closely related concepts of strangeness and hypercharge are introduced. Lots of new particles were discovered in the early '60's. With a lot of work, it was finally determined that $SU(3)$ is the right symmetry group to use to organize them. The main input is the existence of a new quantum number conserved by the strong interactions: *strangeness* S. It is also convenient to use *hypercharge* $Y \equiv B + S$. It is an *observation* that $Q = T_3 + Y/2$. (Q is the electric charge and T_3 is the diagonal I-spin generator.)

  The observed particles of nature appear organized into the $SU(3)$ multiplets **1**, **8**, and **10**. Thus, particles can be labeled by quantum numbers that specify where they appear in an $SU(3)$ irrep. We have seen J and J_3 of $SU(2)$ angular momentum and I and I_3 of $SU(2)$ isospin. Now, in $SU(3)$ flavor, we label the irrep by its highest weight μ and locations in it with weights that are eigenvalues of the Cartan subalgebra $H_1 = T_3 = I_3$ and $H_2 = (\sqrt{3}/2)Y$. You will want to be familiar with the multiplets in the figures 10.8 (or 10.10), 10.11, and 11.31.

- 11.2 Gell-Mann-Okubo formula: This is one of the classic results of the Eightfold Way (another name for the $SU(3)_{flavor}$). This calculation of mass splittings draws on Sec. 10.12. It remains one of the few calculations in elementary particle physics that says something substantive about masses. (Beware that in Eq. 11.19 the T's on the LHS and RHS are not the same.)

  Notice that this approach gets results from symmetry arguments without introducing an underlying quark model. With the benefit of hindsight, such an approach seems seriously wrongheaded. At the time, it wasn't. This preceded the quark model. Even after the quark model arrived, many considered quarks to be just a convenient nemonic to aid in calculation. The last vestiges of such thinking were not eradicated until the November revolution of 1974—the discovery of the charmed quark. Although we all now believe in quarks, we have built a theory QCD which is designed to reproduce the experimental result that an individual quark will never be observed in isolation.

- 11.3 Hadron resonances: *Resonance* is the name for a "particle" that has a very short lifetime ($\sim 10^{-24}$ sec.) due to decay by the strong interaction. In that very short time, the particle does not travel far enough to be observed directly. We know of it only indirectly through its decay products.

The main result here is the mass splitting formula for the **10**. It is the equal spacing rule in 11.34. The prediction of the Ω^- mass is one of the few successful mass predictions ever made.

- 11.4 Quarks: What could I possibly add?

All of this was worked out before the development of and thus without the benefit of what we now call the *standard model*.

Chapter 16

Here we deal with the same group $SU(3)$ but with completely different physics. Each flavor and spin of quark comes in three colors. The transformations that scramble the colors are the group $SU(3)_{color}$. It is the gauge group of QCD. The quarks carry a fundamental **3** of color. At our present level of understanding, this is much more important than flavor because the colors are charges for the force field (with quanta gluons) in QCD that gives the strong interaction. The flavors don't seem to have any apparent purpose.

First there are the physics reasons for introducing color and $SU(3)_{color}$. Then the assertion that we need a rule that restricts the hadrons to be color singlets. To understand where that might come from, there are some very qualitative comments on forces and dynamics and why singlets might be the most strongly bound. The TT form of the interaction form as in Eq. 16.8 fits in the picture.

From a technical perspective, the important point how to construct color singlets from triplet quarks and anti-quarks.

The discussion of mass splittings would fit better back Ch. 11 as an alternative take on the pure group theory appproach.

Chapter 17

This is a very brief discussion of the role of color hyperfine interactions in baryon mass splittings. It's the most non-trivial result from the quark model.

Reading 17, for Thursday, Mar. 13

Selections on SU(5) and SO(10) as GUT gauge groups

Warning: If you have not had 245B or by some other route became familiar with the standard model, these readings may not make much sense.

The minimal read is Sec. 14.1 of Cheng and Li. For my money, it is the best quick introduction to SU(5) as a GUT. If you want to get into the particle physics a bit more, Sec. 14.2 on SSB is the next thing to look at. The other topics like running couplings, proton decay, are baryogenesis are fascinating particle physics but more than we can get into in this quarter.

In Georgi, the heart of Ch. 18 is Sec. 18.6. It covers material similar to Sec. 14.1 of Cheng and Li. Georgi Sec. 18.7 is similar to Cheng and Li Sec. 14.2 in covering SSB.

For a quick introduction to the next step up, see Kaku Sec. 18.7 for the basics of SO(10) as a GUT. And it somehow escaped my attention until just now that the SO(10) chapter near the end of Georgi was greatly expanded from the first edition and now has more about SO(10) as a GUT gauge group. I haven't read it yet.

There is material in Zee, Chapters VII.5, VII.6, VII.7 and, of course, in many other books too.
