Angular distribution in e^+e^- annihilation

When the energy is high enough to ignore the masses, the angular distribution of final state fermions is $(1+\cos^2\theta)$. The relation relies on the fact that eigenstates of helicity and and of $(1\pm\gamma_5)$ are the same for massless Dirac particles. They are not the same for non-zero mass. The electrons and quarks are described by Dirac fields. From our discussion of representations of the Lorentz group, we know that the fermion field is $\psi = \psi_L \oplus \psi_R = \left(\frac{1}{2},0\right) \oplus \left(0,\frac{1}{2}\right)$, and the photon vector field is $A = \left(\frac{1}{2},\frac{1}{2}\right)$. The $\left(\frac{1}{2},0\right)$ part of the fermion field destroys left handed fermions and creates right handed fermions. The conjugate of this part transforms as $\left(0,\frac{1}{2}\right)$ and creates left handed fermions and destroys right handed fermions. Thus the vector part in $\overline{\psi} \otimes \psi$ comes from $\overline{\psi}_L \otimes \psi_L$ and $\overline{\psi}_R \otimes \psi_R$ and not from $\overline{\psi}_R \otimes \psi_L$ or $\overline{\psi}_L \otimes \psi_R$. This means that the fermion antifermion pairs in either the initial or final state have opposite helicity, e.g. spins in the same direction. With the annihilation going through a photon with total angular momentum J=1, the cross section has the form

$\frac{d\sigma}{d\Omega} = \left| T(J,\lambda) d(\theta)^J_{\lambda_a-\lambda_b,\lambda_c-\lambda_d} \right|^2$. In this, we need to fill in J=1 and either +1 or −1 for the helicity differences but not zero. The ds are

$d^1_{11} = d^1_{-1-1} = \frac{1+\cos\theta}{2}$ and $d^1_{1-1} = d^1_{-11} = \frac{1-\cos\theta}{2}$. Along with the fact that we are assuming just lowest order with just one photon, parity, charge conjugation, and rotational invariance tell us that all the contributing Ts are equal in magnitude. Finally spin averaging gives $\frac{d\sigma}{d\Omega} \propto \left|d(\theta)^1_{11}\right|^2 + \left|d(\theta)^1_{1-1}\right|^2 + \left|d(\theta)^1_{-1-1}\right|^2 + \left|d(\theta)^1_{-11}\right|^2 \propto 1 + \cos^2\theta$.