Pion electromagnetic form factor(s)?

The proton has two electromagnetic form factors, F_1 and F_2 or G_E and G_M. A quiz question asked how many electromagnetic form factors the pion has. I claimed that because the pion is spin zero, it can have no magnetic moment, and therefore the answer is that it has just one form factor. This argument was not universally appreciated. Here, I offer a first draft of a more detailed argument. If you see places where it still needs work, let me know. One counterargument was that if the spins in some sense cancel, then since the quarks have different masses and charges, the moments will not.

Let's look at the matrix element of the EM current between pions of momentum p and p′=p+q, i.e. the current carries momentum q.

$$\langle p'|j^\mu(q)|p\rangle$$

One form factor is easy enough to construct. Using p and q and current conservation, we get

$$\langle p'|j^\mu(q)|p\rangle = \frac{1}{2}(p'+p)^\mu F(q^2) = (p+\frac{1}{2}q)^\mu F(q^2) \quad (1)$$

In discussing the proton form factors, we have seen that a magnetic form factor has the form

$$q^\nu \Sigma_{\mu\nu} \quad (2)$$

with Σ antisymmetric. A magnetic moment comes from a non-zero limit for Σ as q goes to zero. So for a pion magnetic form factor, we need an operator of that form with a non-vanishing matrix element

$$\langle p+q|\Sigma_{\mu\nu}|p\rangle$$

We could try the form

$$(p'_\mu p_\nu - p_\mu p'_\nu) = (q_\mu p_\nu - p_\mu q_\nu)$$

but when contracted with q as in (2), it leads back to the form (1). Since we already thought we had exhausted the possibilities in (1), this is no surprise.

However, the idea of the counterargument above is that there is some other vector or tensor lurking in the pion state that can be used in addition to p and q to construct an object of the right form. The problem with this is that it contradicts the statement that the pion is spinless. Our study of the irreducible representations of the Poincare group told us that states are labeled by mass and spin and that's it so far as anything with non-trivial spacetime properties goes. Thus if the pion is spinless, there is nothing else to work with.

At least in the pion rest frame, we can make the argument more concrete and show that the pion can have no static moment. Let $\mathcal{R}$ be the representations of rotations on the Hibert space of pion states and let R be the representation on 3d vectors. Work in the pion rest frame where p=(m,0,0,0).

$$\langle m|\Sigma_{0i}|m\rangle = \langle m|\mathcal{R}\mathcal{R}^{-1}\Sigma_{0i}\mathcal{R}\mathcal{R}^{-1}|m\rangle = \langle m|\mathcal{R}^{-1}\Sigma_{0i}\mathcal{R}|m\rangle = \langle m|\Sigma_{0j}R_i^j|m\rangle$$

Similarly

$$\langle m|\Sigma_{ij}|m\rangle = \langle m|\Sigma_{kl}R_i^k R_j^l|m\rangle$$

Since the rotation is arbitrary, it must be that

$$\langle m|\Sigma_{\mu\nu}|m\rangle = 0.$$

How do we understand this result in the context of the original objection? The statement of vanishing spin along the 3-axis is

$$({}_{\bar{q}}\langle\uparrow|_q\langle\downarrow| - {}_{\bar{q}}\langle\downarrow|_q\langle\uparrow|)(J_{q3}+J_{\bar{q}3})(|\uparrow\rangle_q|\downarrow\rangle_{\bar{q}} - |\downarrow\rangle_q|\uparrow\rangle_{\bar{q}}) = 0$$

If we assume the quark moments are multiples of the spins, the cancellation is slightly less direct, but we still get zero.

$$({}_{\bar{q}}\langle\uparrow|_q\langle\downarrow| - {}_{\bar{q}}\langle\downarrow|_q\langle\uparrow|)(aJ_{q3}+bJ_{\bar{q}3})(|\uparrow\rangle_q|\downarrow\rangle_{\bar{q}} - |\downarrow\rangle_q|\uparrow\rangle_{\bar{q}}) = 0$$

The last result begs us task about the magnetic susceptibility or polarizability of the pion. That involves the square of the field, *i.e*. two currents.

$$\langle p'|j^\mu(q_1)j^\nu(q_2)|p\rangle$$

I have not thought about this before. I'm guessing that the piece we want is something like

$$q_{1\alpha}q_{2\beta}\langle p|\Sigma^{\mu\alpha}\Sigma^{\nu\beta}|p\rangle$$

Has anybody done the Stern-Gerlach experiment or Compton scattering with pions?