

Color singlets have integer electric charge:

Singlets are found only in the decomposition of reps. with triality zero:

- 1) The only invariants of SU(3) are δ_j^i , ϵ^{ijk} , and ϵ_{ijk} .
- 2) All the irreps are found in the decomposition of the products of the fundamentals. In the case of SU(3), there are two fundamentals, 3 and 3^* .
- 3) Start with an arbitrary product of m factors of 3 and n factors of 3^* . The tensors that carry the rep. have components with m indices of 3 type and n of 3^* type. A singlet has no indices. To find a singlet (if it exists), we can contract with the invariants to try to get down to a non-vanishing tensor with no indices, i.e. a singlet.
- 4) Using δ_j^i reduces both m and n by one, while using ϵ^{ijk} , or ϵ_{ijk} reduces either m or n by three. Thus in both cases the triality = $m-n \pmod 3$ does not change. Since $m=n=0$ for the singlet, its triality is zero, and it can be found only in a product that has triality zero.

If the triality is zero, then the electric charge is integer multiples of the proton charge:

Let m_U be the number of up type quarks (u,c,t), and let m_D be the number of down type (d,s,b) quarks. For the anti-quarks, use n_U and n_D . Also write $m=m_U+m_D$ and $n=n_U+n_D$. Then in units of the proton charge, the charge of a combination of quarks is

$$\begin{aligned} Q &= (2/3)m_U - (1/3)m_D - (2/3)n_U + (1/3)n_D \\ &= (2/3)(m_U+m_D) - m_D - (2/3)(n_U+n_D) + n_D \\ &= (2/3)(m-n) - m_D + n_D, \end{aligned}$$

which is integer for triality zero.