Beats

Beats are an interesting wave interference effect. They can be perceived directly for sound waves. musicians are familiar with them. When two notes that are very close but not quit the same are played at the same time, a slow modulation of the loudness of the sound is heard.

One source generates a tone with frequency f_1 and a second with f_2. We suppose that the two frequencies are close, so that the difference is much smaller than the average. The superposition principle tells us that the resulting wave is just the sum of the two waves that would be there if only one or the other source were present. For simplicity, I will give them the same amplitude and phase and let the fixed point of observation be the origin x=0. To avoid confusion between f for disturbance and f for frequency, I will use F for disturbance here.

$$F = F_1 + F_2 = A \sin(2\pi f_1 t) + A \sin(2\pi f_2 t).$$

This does not say much of anything yet. But let's rewrite it using the identity

$$\sin A + \sin B = 2 \sin(\{A+B\}/2) \cos(\{A-B\}/2).$$

That gives

$$F = 2 A \cos(2\pi\{f_1 - f_2\}t/2) \sin(2\pi\{f_1+f_2\}t/2)$$

So what? Well, let's think about it. Remember that the two frequencies are very close. Thus the first cos factor changes slowly with t while the second oscillates with the average frequency, which is close to both of the original frequencies. This tone is slowly modulated in amplitude by the first cos factor. It slowly changes from 1 to 0 to -1 to 0 to 1 and so on. Whenever it is near 1 or -1, the sound is loud. When it is near zero the sound is soft. The time it takes to go from loud to soft to loud is

$$t_B = 1/|f_1 - f_2|.$$

Thus the frequency of the cycle from loud to loud is

$$f_B = 1/t_B = |f_1 - f_2|.$$

This is the beat frequency.