Information, entropy, and black holes

Entropy and the Second Law of thermodynamics arose from practical studies of engines that were important to the industrial revolution. Similarly a quantitative theory of information arose from practical studies of electronic communications in an information-based economy as those became increasing important about 60 years ago. It is interesting that there is a very close relationship between the quantitative descriptions of entropy and information.

Information

Here is a *very* brief description of the quantitative definition of information. The discussion is carried out relative to an ensemble of possible states or messages E. The qualitative idea is that if E has many states or many possible messages, then specifying a particular state gives a lot of information. If there are only a few possibilities then indicating one gives less information. In the limit of only one possible state, being told what state the system is in gives no information at all because you already know.

How many bits does it take to indicate a particular state? For one state, zero bits, for two states, one bit, for four states, two bits, and in general for 2^N states, N bits. This motivates the definition that the information in a message that is one out Q possible messages or in a message that selects one of Q possible states is

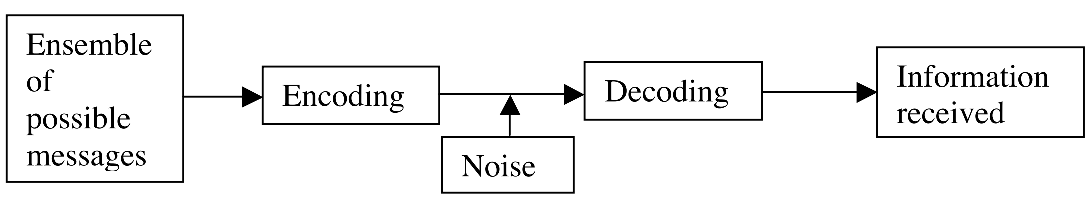
$$I = \log_2 Q$$

Evidently, this definition is closely related to the statistical mechanical definition of entropy $S = \ln w$.

How much information is in a message? If the message narrows the possibilities from $Q_1 = 2^N$ to $Q_2 = 2^M$, the information is the message is

$$\log_2 Q_1 - \log_2 Q_2 = N - M.$$

The general communications setup is indicated in the diagram



This concept is used to analyze things like the information carrying capacity of communication channels, the efficiency of encoding methods, etc. Lossy encoding (e.g. JPEG and MP3) or the introduction of noise decrease the information in a message. Thus while entropy increases, information decreases. Thus information is sometimes called negentropy.

Black hole information problem

Black hole basics

There is a formal analogy between the properties of a black hole and thermodynamics. A black hole has a singularity at a point inside of it where the curvature of space becomes infinite, i.e. the gravitational field is infinite. The interior region of the black hole is separated from the exterior region by a spherical surface called the event horizon. Matter and radiation can enter the black hole by crossing the event horizon from the outside to the inside, but nothing can escape. Anything that crosses the horizon cannot avoid falling into the singularity and being crushed there. For a black hole of mass M, the event horizon has radius $R \propto M$ and area $A \propto R^2 \propto M^2$. The acceleration of gravity at the horizon is $GM/R^2 \propto 1/R$. Since matter and energy can enter the black hole, but nothing can leave, the mass M of the black hole always increases—at least classically.

No hair theorem

There is a famous and deep theorem that says black holes have no hair. What this means is that a black hole is completely specified by its mass, charge, and angular momentum. Since we are not going to be dealing with charge or angular momentum, we can over-simplify a bit and say that all black holes of mass M are the same. There is no way to tell from the outside what matter went into the black hole to form it or what subsequently fell in. There is no structure (hair) on the surface (horizon) that is a trace of what went in. All that detailed information is lost.

Thermodynamics

If a black hole were in thermal interaction with its environment, energy could flow into the black hole but not out. Thus the black hole would act like it had zero temperature and infinite heat capacity. There could be equilibrium at T=0 only. This seams unsatisfactory. Fortunately, it is not the whole story. Recall that the specific heats of ideal gases and black body radiation could not be properly described at the classical level. Quantum mechanics came to the rescue. Here too it is crucial.

Suppose that in a quantum fluctuation, a pair of particles is created just outside the horizon. As viewed by an observer far from the black hole, these particles have negative potential energy. Give one of the particles enough kinetic energy to make the total energy of the pair zero. Now the particle without the kinetic energy can fall into the black hole while the other particle escapes to infinity. When the negative energy particle goes into the black hole the black hole mass *decreases* and a particle is radiated away while total energy is conserved. This radiation was first described by Hawking and Bekenstein. A detailed and complicated calculation reveals that the radiation has the spectrum of a body at temperature $T \propto 1/M$. This is amazing. Now it is not too far fetched to speculate that there is the following dictionary for the analogy:

U	⇔	M
T	⇔	$1/R \propto 1/M$
S	⇔	$A \propto R^2 \propto M^2$
$dS/dU = 1/T$	⇔	$dM^2/dM \propto M \propto 1/T$

Thus the black hole acts like it has a temperature $T \propto 1/M$. It radiates energy and thus mass and eventually disappears. We learned that the rate of radiation is proportional to $T^4 \propto 1/M^4$. So this is initially very slow but speeds up at the end just before the black hole disappears in a flash. As this happens, the mass of the black hole and thus its entropy are decreasing. However, since there is compensating entropy in the radiation, this does not violate the Second Law.

For systems that we have studied, the entropy is proportional to the volume of the system in the sense that if we have, for example, a box of gas with entropy S and then we double the volume of the box and double the number of particles, the entropy doubles to 2S. Notice however, the very curious aspect of the black hole entropy that it is proportional to the surface area of the black hole not to its volume.

In our discussions of entropy, we began with a somewhat abstract thermodynamic approach but then gave a more intuitive account in terms of statistical mechanics and counting states. It remains an open question how to get the black hole entropy from a statistical mechanical counting of states. Some believe that string theory may provide an answer, but it's not there yet.

Information loss?

There is a long-standing, unresolved question about black holes and information. There are many ways to state it. Here is one. At the end of the quarter, all the physics students in the universe toss their physics books into the same pile. Although you may have some doubts, these books contain a lot of information and are in a state of low entropy. If there are enough books, they will collapse into a black hole. Considering the no hair theorem, it appears that the information has been lost. Worse, the black hole will eventually disappear leaving behind a lot of thermal radiation and (apparently) very little information. Thus one begins in a single zero temperature state (the pile of books) with lots of information and no entropy and ends in a state (the radiation) with non-zero temperature and lots of entropy. Ordinary quantum mechanics does not provide a mechanism for such a transformation. Is quantum mechanics wrong or at least incomplete? Is the final state not really high entropy but actually has all the physics from the books some how encoded in it? This remains an open question. Even those who are adamant that the information is preserved have not been able to present a convincing mechanism by which that happens.