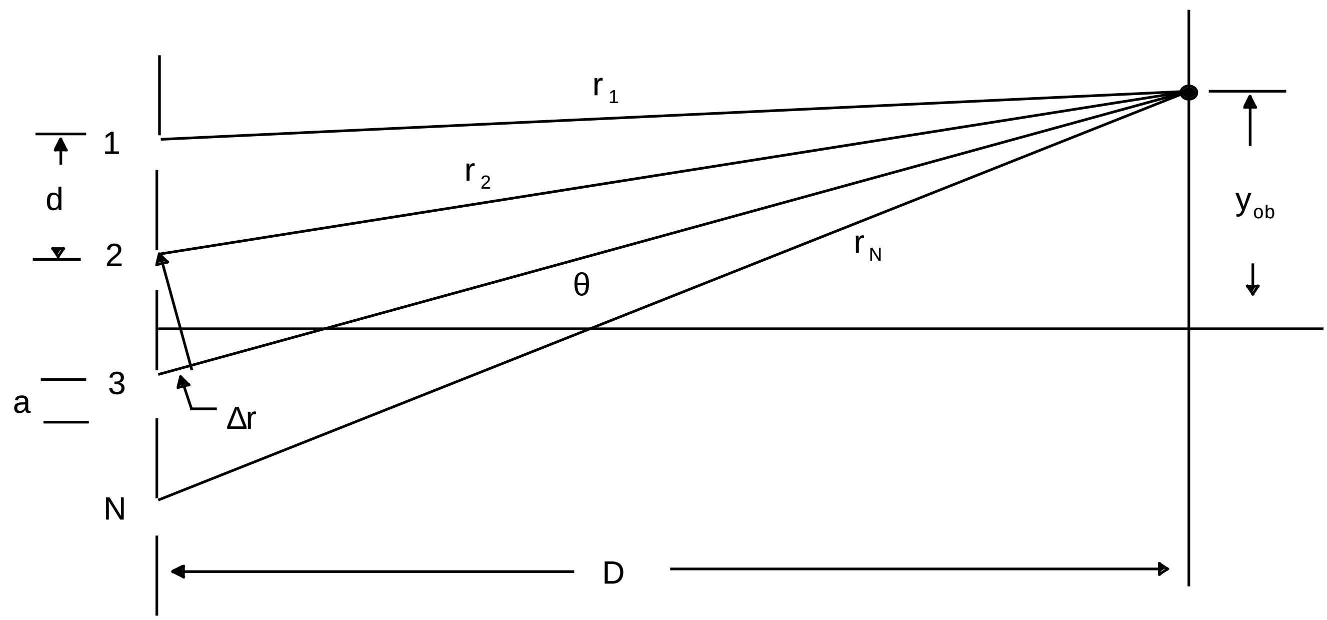
Multiple sources with a finite width: interference and diffraction together

In any real, non-idealized example of interference, there is likely to be diffraction at work too since the sources have a finite size. Unless that size is smaller than one wavelength, each source will produce a diffraction pattern. How do we put this together? Answer: (You guessed it.) "It's the superposition principle!"

So now let's suppose we have a situation like we did for multiple source interference but also account for the finite size of each source.

The separation between the slits (center to center) is d, and the width of each is a. As before, $\Delta r = d \sin\theta$ and $\Delta\phi = k\,\Delta r = k\,d \sin\theta$, $\beta = ka \sin\theta$, and for simplicity $y_{ob}/D \ll 1$,



$\theta \ll 1$, $Nd \ll D$.

To get the wave at the observation point, we must add the waves from each of the N sources.

$$y = y_1 + y_2 + y_3 + \ldots + y_N$$

But now what is the wave at the observation point from each of the finite with sources? Looking back at the diffraction derivation (where I have now added another line for the amplitude of the wave), we see that

$$y_1 = A \cos(kr_1 - \omega t + \beta/2)\ \sin(\beta/2)/(\beta/2)$$

$$y_2 = A \cos(kr_2 - \omega t + \beta/2)\ \sin(\beta/2)/(\beta/2)$$

In general for the n-th term,

$y_n = A \cos(kr_1 - \omega t + \beta/2 + \{n-1\}\Delta\phi) \sin(\beta/2)/(\beta/2)$ so that

$y = A\, \Sigma_0^{N-1} \cos(kr_1 - \omega t + m\Delta\phi) \sin(\beta/2)/(\beta/2)$

$\quad = A \sin(\beta/2)/(\beta/2)\, \Sigma_0^{N-1} \cos(kr_1 - \omega t + m\Delta\phi)$

At this point, we can see what is going to happen. The diffraction piece $(\sin(\beta/2)/(\beta/2))$ is a common factor, and the sum is the same one we had for multiple source interference. Thus we are going to get a result that is the *product* of the diffraction and the interference results. Fortunately we have already done the sum, so using the result from *sum formula* again gives

$y = A \sin(\beta/2)/(\beta/2) \cos(kr_1 - \omega t + \{N-1\}\Delta\phi/2) \sin(N\Delta\phi/2)/\sin(\Delta\phi/2)$.

Now we just apply the same reasoning that we used in the past to get the average intensity

$I_{avg}(\theta) = \{I(0)/N^2\} [\sin(\beta/2)/(\beta/2)]^2 [\sin(N\Delta\phi/2)/\sin(\Delta\phi/2)]^2$.

This is the intensity pattern for N finite sources. You should try to figure out what this looks like. We will spend some time in class understanding this result. (It is used without derivation in Sec. 17.3.4 of the text.)