

Initial condition problem

At $t=0$, initial conditions:

$$y(x,0) = Ae^{-x^2/L^2} \quad \frac{\partial y(x,0)}{\partial t} = 0$$

What is evolution in time?

$$y(x,t) = f(x-vt) + g(x+vt)$$

$$y(x,0) = f(x) + g(x)$$

$$\frac{\partial y(x,0)}{\partial t} = v[-f'(x) + g'(x)] = 0 \quad f' = g'$$

$$f = g \quad (\text{with adjustment of integration constant})$$

$$y(x,0) = 2f(x) = Ae^{-x^2/L^2}$$

$$f(x) = g(x) = \frac{A}{2}e^{-x^2/L^2}$$

$$y(x,t) = \frac{A}{2}e^{-(x-vt)^2/L^2} + \frac{A}{2}e^{-(x+vt)^2/L^2}$$