

Steps of the Stirling cycle:

- 1) Isothermal expansion from V_A to V_B at T_H . Q₁ from the hot reservoir is converted to work W₁.
- Isochoric cooling from T_H to T_C. There is heat flow from the working substance (ideal gas) to the cold reservoir as the working substance cools.
- 3) Isothermal compression from V_B to V_A at T_C . Work done on the gas results in heat flow to the cold reservoir.
- Isochoric heating from T_C to T_H. The temperature of the working substance is raised by heat flow Q₄ from the hot reservoir.

The total heat flow into the engine from the hot reservoir is Q_1+Q_4 . The net work done by the engine is W_1+W_3 with $W_1 > 0$ and $W_3 < 0$.

Thus the efficiency is $e = (W_1+W_3)/(Q_1+Q_4)$.

Using previous results for work in an isothermal process,

$$\begin{split} W_1 + W_3 &= nRT_H \ln\left(\frac{V_B}{V_A}\right) + nRT_C \ln\left(\frac{V_A}{V_B}\right) = nR(T_H - T_C) \ln\left(\frac{V_B}{V_A}\right). \\ Q_1 &= W_1 = nRT_H \ln\left(\frac{V_B}{V_A}\right). \end{split}$$

In the isochoric compression of step 4 there is no work and

$$Q_4 = \Delta U = nR\frac{q}{2} (T_H - T_C)$$

Then it is just some algebra to obtain

$$e_{S} = \frac{W_{1} + W_{3}}{Q_{1} + Q_{4}} = \frac{T_{H} - T_{C}}{T_{H} + X} \text{ with } X = \frac{q}{2} \frac{T_{H} - T_{C}}{\ln\left(\frac{V_{B}}{V_{A}}\right)}.$$

Since X is positive, $e_s < \frac{T_H - T_C}{T_H} = e_c$ i.e. the Stirling efficiency is less than the Carnot

efficiency. Why is that? Is the Stirling cycle reversible? In particular, are the two isochoric processes reversible? Is placing a gas at T_H in contact with a reservoir at T_C and letting it cool reversible?