PHASE SUM
This is a proof of the formula that was used in the derivation of the N-source interference intensity formula. The result is

\[ \sum_{n=0}^{N-1} \cos(A + n\phi) = \cos\left(A + \frac{N - 1}{2} \phi\right) \frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}. \]

To make the proof simple, it is best to use a complex representation \( e^{i\theta} = \cos \theta + i \sin \theta \).

The other thing to use is the partial geometric sum

\[ \sum_{n=0}^{N-1} x^n = \frac{1 - x^N}{1 - x}. \]

The sum we want is the real part of the sum

\[ \sum_{n=0}^{N-1} e^{i(A + n\phi)} = e^{iA} \sum_{n=0}^{N-1} (e^{i\phi})^n = e^{iA} \frac{1 - e^{iN\phi}}{1 - e^{i\phi}} = e^{iA} e^{iN\phi/2} \left( e^{-iN\phi/2} - e^{iN\phi/2} \right) = e^{iA} e^{iN\phi/2} \left( e^{-i\phi/2} - e^{i\phi/2} \right) = e^{i\left(A + \frac{N-1}{2} \phi\right)} \frac{N\phi}{\sin \left(\frac{\phi}{2}\right)}. \]

‘Not too hard.'