PHASE SUM

This is a proof of the formula that was used in the derivation of the N-source interference intensity formula. The result is

$$\sum_{n=0}^{N-1} \cos(A + n\phi) = \cos\left(A + \frac{N-1}{2}\phi\right)\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)} \quad .$$

To make the proof simple, it is best to use a complex representation

$$e^{i\theta} = \cos\theta + i\sin\theta \quad .$$

The other thing to use is the partial geometric sum

$$\sum_{n=0}^{N-1} x^n = \frac{1-x^N}{1-x} \quad .$$

The sum we want is the real part of the sum

$$\sum_{n=0}^{N-1} e^{i(A+n\phi)} = e^{iA}\sum_{n=0}^{N-1}\left(e^{i\phi}\right)^n = e^{iA}\frac{1-e^{iN\phi}}{1-e^{i\phi}} =$$

$$e^{iA}\frac{e^{iN\phi/2}\left(e^{-iN\phi/2} - e^{iN\phi/2}\right)}{e^{i\phi/2}\left(e^{-i\phi/2} - e^{i\phi/2}\right)} = e^{i\left(A+\frac{N-1}{2}\phi\right)}\frac{\sin\frac{N\phi}{2}}{\sin\frac{\phi}{2}} \quad .$$

‘Not too hard.