Introduction to waves

General remarks

We are bathed in waves, and two of our senses are wave detectors. We hear via sound waves, and we see via electromagnetic waves in the frequency range we call light. In taking a bath or going for a swim, we are literally bathed by water waves. To produce the sound waves we hear as music, a violin string has a displacement wave moving on it, and similar statements hold for other instruments.

So waves are a very general phenomena. And yet in a sense, they are not microscopically fundamental. Just as there is no notion of temperature for a single molecule but only for a gas of many molecules, so too, a single molecule of violin string does not make a wave. It is the coordinated movement of all the molecules of the string that is the wave. So waves like temperature are *emergent* phenomena. They are not manifest at the most microscopic level, but appear at a higher level when there are many particles or a macroscopic system.

Note that when waves on a string, waves in water, or sound waves travel along, it is the disturbance in the medium that travels not the medium itself. When a wave travels down a string, each piece stays right where it is and just moves back and forth a little while the wave travels the whole length of the string. Although the waves you might surf off the California coast may have been generated by a storm in the Sea of Alaska, no water molecule made that trip with the wave. But with the rising and falling of the water wave as it travels thousands of miles, something does come all that way: *energy*. No one who has been driven into the sand by a breaking wave can doubt that waves transport energy.

So let us define a wave as a traveling disturbance in a medium. Right off we need to make a distinction. Let’s refer to waves in real physical media like water or air or the earth (seismic waves) as *mechanical waves*. Distinct from such mechanical waves is light. As we discussed at some length last quarter, electromagnetic (EM) waves do not travel in a medium with ordinary properties. For many purposes, it is good to avoid thinking about there being any medium associated with EM waves. But if you do wish to associate a medium with EM waves you can call it the vacuum or the electromagnetic field. This medium has very definite properties, but as we saw they are very strange and not at all like water or air or rock. So if we keep in mind that qualification about the strangeness of the medium of EM waves, then it is OK to use the definition above.

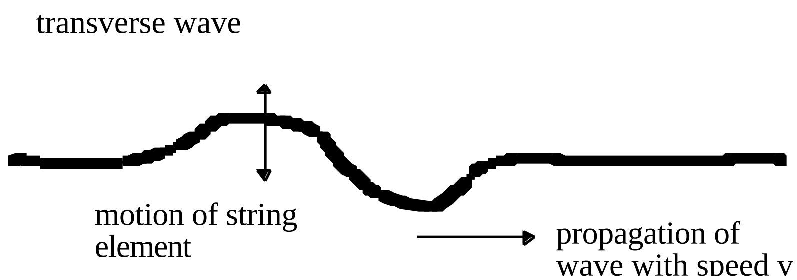
But why do disturbances travel in a medium at all? It is because nearby elements of the medium interact with each other. If one little section of a string is moved away from its equilibrium position, it exerts a force on those nearby elements. Those move a little and then exert a force on their neighbors, and so it goes. It's such a simple and general mechanism that we can expect to find waves in many circumstances, and indeed we do.

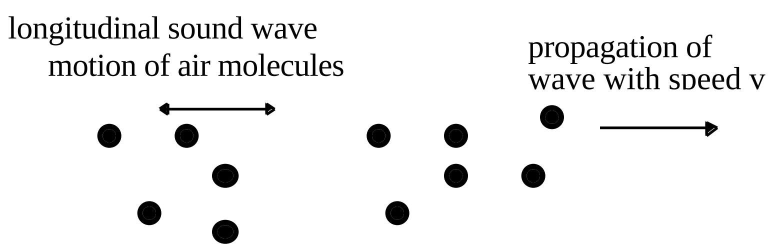
Characteristics of waves

Waves are described by several qualitative terms and quantitative variables.

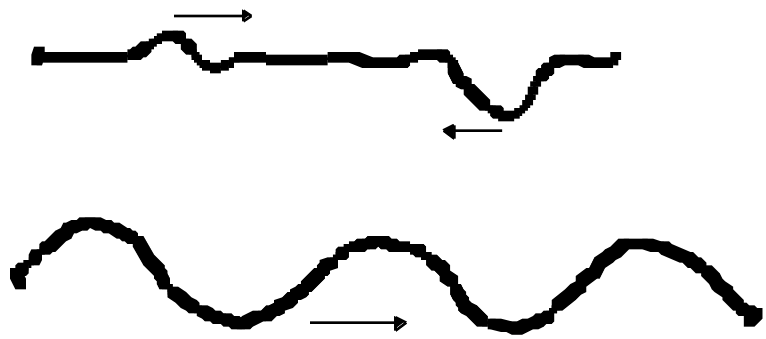
The disturbance in the medium travels with a characteristic *speed* v. For most of the cases we will discuss the speed is a characteristic of the medium and is independent of other properties of the wave. For example, in vacuum, all light waves travel with speed $v=c$, independent of the wavelength or intensity of the light. If the amplitude is not too big, all waves on a string travel with the same speed. The speed depends only on the properties of the string (density and tension). However, there are some important cases where that is not the case. For example when light travels through a medium like water or glass, different wavelengths travel at slightly different speeds. That is where rainbows come from and why a prism can separate white light into its component colors. Waves on the surface of water also travel at different speeds depending on wavelength.

The disturbance can be a movement of the medium in a direction perpendicular to the direction in which the wave is traveling, e.g. a wave in a string. In this case, we say that it is a *transverse wave*. Alternatively the elements of the medium may move back and forth along the line that is the direction of the wave propagation. That is a *longitudinal wave*. An important example of that is sound in air. As the disturbance that is the sound moves across the room, individual molecules move a very small distance in the direction of propagation and then back. Surface water waves are again an exception in combining both transverse and longitudinal motion.



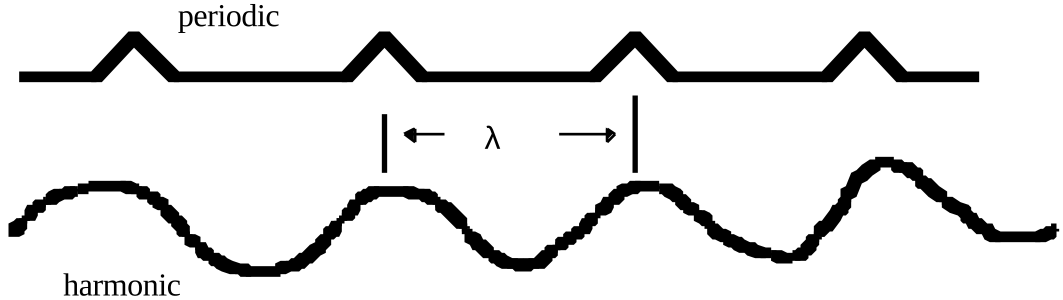


The *amplitude* of a wave gives how large the displacements of the medium are as the wave goes by.
The shape of a wave can be just about anything. Two extremes are *pulses* and *periodic waves*.



On the top, there is a small amplitude pulse moving to the right and a larger amplitude pulse moving to the left. On the bottom, there is a periodic wave moving to the right.

Since *periodic and harmonic waves* are so important, let's look more closely at them. A harmonic wave is a special case of a periodic wave. A periodic wave has a pattern that repeats itself in space and in time. If you take a snapshot of the wave at some time and then if you can move the picture over by some distance λ and have it look just like it did before, then it is a periodic wave. Similarly if a picture of the wave at time t and at time t+T look the same, then the wave is periodic. Harmonic waves are the case of periodic waves when the shape is given by a trig function cos or sin or a combination of those.



A mathematical description can also be given. For a wave propagating in the x direction, let $f(x,t)$ be the function that describes the displacement associated with the wave. For the case of the string, it would be how far at time t a little piece of the string at position x is displaced in the transverse direction y from its equilibrium position. Then if the wave is periodic,

$$f(x-\lambda,t) = f(x,t) \quad \text{and} \quad f(x,t-T) = f(x,t)$$

An example of a harmonic wave is

$$f(x,t) = A \sin(2\pi x/\lambda - 2\pi t/T)$$

This wave has amplitude A, wavelength λ, and period T. Since a shift in x by Δx has the same effect as a shift in time of $\Delta t = - \Delta x\, T/\lambda$, the speed of the wave is $v = \lambda/T$, and it is moving to the right in the direction of increasing x. (To understand those claims, draw yourself a couple little pictures of the wave at times t and $t+\Delta t$, and you will see what is happening.)

Here is some more notation:
Wave number $k = 2\pi/\lambda$.
Frequency $f = 1/T$. (Don't confuse this f with the displacement $f(x,t)$.)
Angular frequency $\omega = 2\pi f = 2\pi/T$.
$v = \lambda/T = \lambda f = \omega/k$.
With those, we can then also write $f(x,t) = A \sin(kx - \omega t)$.

A couple more comments:
A wave moving to the left has the form $f(x,t) = A \sin(kx + \omega t)$.
There is nothing special about the sin function, we could just as well use the cos, or any linear combination of the sin and the cos with the same argument or $\sin(kx - \omega t + \theta)$. While all the choices are very similar, they are not exactly the same. In what ways are they the same, and in what ways different?