Final Physics 9HB winter 2001

Use a law blue book. Closed book. Two 3”x5” note cards. PRINT your name on your blue book. To get credit, you must show your work. Be neat, clear, and organized. If we can’t read it or figure it out, we can’t give you any points. Also be sure to give your answers with appropriate units.

\[ k_B = 1.38 \times 10^{-23} \text{J/K}, \quad N_A = 6.02 \times 10^{23}/\text{mole}, \quad R = 8.31 \text{ J/(mole K)} \]

1. (20) A 1m (1m (1m box holds 40 moles of helium, an ideal gas with \( q=3 \) degrees of freedom and a mass per atom \( m=4g/N_A \). The pressure is \( 10^5 \text{ N/m}^2 \). The box of gas is in thermal equilibrium with a cubic meter of water.
   a) What is the temperature of the water?
   b) What is the average translational kinetic energy of a water molecule?
   c) What is the rms speed of a helium atom?
   d) The box is removed from contact with the water, insulated and is now surrounded by vacuum. One wall is slid to the side so that the box is open to the vacuum for 1 ms and then slid closed again. After a short time, the remaining molecules come to equilibrium. Are the gas molecules that remain in the box and the water still in thermal equilibrium? Explain.

2. (25) To make things a little simpler, let’s suppose that air is 50% \( \text{O}_2 \) and 50% \( \text{N}_2 \) by number. Consider a small box that is divided in half. On the left half there is one \( \text{O}_2 \) molecule, and on the right half one \( \text{N}_2 \) molecule. The box is in thermal equilibrium with air at 300K. The partition between the two halves of this small box is removed for a few seconds and then replaced.
   a) What is the probability that the left half now has one \( \text{O}_2 \) and the right one \( \text{N}_2 \)?
   b) Using our simplified air mixture, suppose that a partition were to drop from the ceiling and divide this room in half. Is there any appreciable probability that the left half contains all \( \text{O}_2 \) and the right all \( \text{N}_2 \)? Explain your answer carefully including an explicit statement of any physical principle that you use. Include comments
on why your answer here differs from your answer to part a (if it does).
c) We saw that the entropy of an ideal gas has the form
\[ S = k_B N \ln \left[ \frac{CV}{N} \right] \]
with \( C \) independent of \( U \). Estimate the ratio of the
probability of finding all \( O_2 \) on the left side and all \( N_2 \) on the right to
the probability of the actual macrostate. Use one mole each of \( N_2 \)
and \( O_2 \).

3. (45) One mole of a monatomic ideal gas \( (q=3, \gamma = 5/3) \) is initially at
\( T_1 = 300K \),
\( P_1 = 10^5 \text{ N/m}^2 \), and \( V_1 \). All steps are quasistatic. In step 1, the gas is
insulated, and it expands until the temperature is \( T_2 = 200K \) and \( V_2 \).
In step 2, it is in equilibrium with a heat reservoir at 200K, and it is
compressed back to its original volume. Finally in step 3, in contact
with (but not necessarily in equilibrium with) a reservoir at 300K, it
is heated at constant volume until it is in equilibrium with the 300K
reservoir.
a) Draw the PV diagram (Is this a closed cycle?) and calculate \( V_1 \)
and \( V_2 \).
b) Calculate the entropy change for the gas for each step. What is
the net entropy change for the gas for the three steps combined?
Explain your answer.
c) Calculate the entropy change of each of the two reservoirs and
the combined entropy change for the two reservoirs?
d) What is the total entropy change of the universe for one cycle?
e) If a Carnot cycle operated between the same two reservoirs,
what would the total entropy change of the universe be for one
cycle?
f) Explain why your answers to d and e are either the same or
different.

4. (over)
4) (10) The present cosmic microwave background radiation (CMB) temperature is 2.7K. At the time the radiation was emitted, the temperature was 3000K. The present value of the Hubble parameter is $H_0 = 1/(14 \times 10^9 \text{ yrs})$. Let $t_0$ be the present time and $t_1$ be the time when the CMB was emitted.

a) By what factor has the universe expanded since time $t_1$?

b) For the period since $t_1$, we can consider two cases. In the first, the universe is matter dominated and $R(t) = R_0 (t/t_0)^{2/3}$ with $t_0 = 2/(3 H_0)$. In the second case, it is dominated by vacuum energy and $R(t) = R_0 \exp[H_0 (t-t_0)]$. Both cases are arranged so that $R(t_0) = R_0$ and $H(t_0) = H_0$. In which case is the time difference $t_0 - t_1$, between the present and the time of CMB emission shorter? Explain.