More cosmology

This is a summary of the results from the discussion in lecture about the evolution of the universe.

- Scale factor for the universe: R(t). Physical separations are proportional to R.
- Notation: $\dot{R} \equiv \frac{dR}{dt}$
- Hubble parameter: $H \equiv \dot{R} / R$
- Hubble expansion: v=Hd
- Energy density: ρ
  - radiation dominated: $\rho \propto R^{-4}$
  - matter dominated: $\rho \propto R^{-3}$
  - curvature dominated: ρ is negligible relative to k/R^{-2}.
  - vacuum or cosmological constant dominated: ρ is constant.
- Friedmann equation (solve to get evolution R(t)):

  $$H^2 - \frac{8\pi G}{3}\rho = -\frac{k}{R^2}$$

  This equation describes physics that is analogous to the problem of throwing an object away from the surface of the earth. The conservation of energy determines the outcome. The results for R(t) below are not big mysteries. They come right out of this equation, and the basic physics can be understood by comparing with the analogous problem just mentioned.
- k=+1, 0, -1
- Present value of the Hubble parameter: $H_0=h_0/(9.78\times10^9$ years)
  $h_0=0.7\pm0.1$
- Critical density: $\rho_c = \frac{3H^2}{8\pi G}$
- Present value: $\rho_{c0} = 1.05 h_0^2 10^4 \, eV/cm^3$
- At early times (less than about 100,000 years), the radiation density dominates, and the Friedmann equation is integrated to give $R \propto t^{1/2}$
- After that, matter becomes most important and $R \propto t^{2/3}$
- Then the value of k becomes important.
  - k=0: The universe is flat. The expansion continues as $R \propto t^{2/3}$.
  - k=+1: The spatial universe is a closed, finite, 3-sphere with positive curvature. It reaches a maximum R and then recontracts to R=0.

  - k=-1: The spatial universe is an open, infinite, 3-space with negative curvature. For large t when the curvature term, $-k/R^{-2}$, dominates, R=t.
- There is also the special case when the energy density is constant. This comes from some kind of vacuum energy or from the cosmological constant, which is just another name for the same thing. If it is very small, it will eventually take over since it is a term in the Friedmann equation that does not decrease with time. The other important possibility is that it was once very large but is now zero. This leads to *inflation*. In either case, the Friedmann equation gives

  $R \propto e^{\overline{H}t}$ with a constant Hubble parameter $\overline{H} = \left(\frac{8\pi G\rho}{3}\right)^{1/2}$.
- So everything depends on ρ_0 relative to ρ_{c0}. In discussing this, it is convenient to use the ratio $\Omega \equiv \frac{\rho}{\rho_c}$ and its present value Ω_0.
  - At the present time,
    - Ω_{CMB} is tiny.
    - The contribution from luminous matter like stars and everything else that we have been able to observe directly is small $\Omega_{LUM} \leq 0.01$. This comes from observation.
    - On the other hand, the nucleosynthesis calculations say that the contribution from ordinary matter is $\Omega_B \approx 0.03$. This is the first dark matter problem: Where is the ordinary matter that is in Ω_B but has not been detected and counted in Ω_{LUM} ?
    - From observations of the rotations and orbits of galaxies, one can figure out, by using Newton's laws, how much gravitating mass there must be around to give those trajectories. That gives $\Omega_M \geq 0.3$. This is the second dark mater problem: What is the gravitating matter in Ω_M that is not the ordinary matter in Ω_B ?
    - Finally, observe that if there has been a period of inflation in the past where R made a huge increase with ρ constant, then the curvature term ($\propto R^{-2}$) became negligible relative to the constant energy density. So when inflation ended, the H^2 and the energy terms in the Friedmann equation were equal to very high accuracy. Thus, inflation predicts $\Omega_0 = 1$. If that is correct, then there is another dark matter problem: What and where is the stuff that fills in the difference between $\Omega_M \approx 0.3$ and $\Omega_0 = 1$?
  - The simple relation v=Hd is only approximately correct for small distances. To see that, ask what H should go in there. Should it be H_0 or H at the time the light was emitted? Neither is correct. You have to account for the fact that there is continuous expansion while the light is traveling from the galaxy to us. Let t_0 be the present time and t the time of emission. If this difference is not

too big, we can expand

$$R(t)= R(t_0) + (t-t_0)\dot{R}(t_0) + \frac{1}{2}(t-t_0)^2\,\ddot{R}(t_0) + \ldots$$

$$= R(t_0)\left[1 + (t-t_0)H_0 + \frac{1}{2}\frac{\ddot{R}(t_0)}{R(t_0)H_0^2}\left\{(t-t_0)H_0\right\}^2 + \ldots\right]$$

The coefficient appearing in the last term is essentially what is called the deceleration parameter

$q_o \equiv -\dfrac{\ddot{R}(t_0)}{R(t_0)H_0^2}$, which tells you about how the Hubble parameter is changing with time. If gravity is slowing the rate of expabsion, then the second time derivative of R is negative and q_0 is positive. After a long song and dance,
one finds an approximate relationship between the Hubble parameter, the distance, and the redshift

$H_0 d = z + \dfrac{1}{2}(1-q_0)z^2$. Observations of z and d allow H_0 and q_0 to be determined. However, it is hard, and q_0 is even less precisely known than H_0. Very new data, recently covered in the news, used supernovas to get a new number for q_0. It was negative so that $\ddot{R}(t_0) \geq 0$! What does this mean? You can put the forms for R(t) that we have in the definition of q_0. That reveals that a critical universe has q_0=1/2. Further, the only case that gives negative q_0 is an exponentially growing universe driven by a cosmological constant!

Conclusion

This discussion of cosmology shows that basic physical principles, discovered here on the earth, can be applied to universe as a whole to understand its behavior. This is an impressive achievement. It is an illustration of one of the the most important aspects of physics, namely its generality. From a small number of fundamental laws, a great deal can be understood.