Lorentz Transformation and Lorentz Contraction

The analytical centerpiece

Hendrik Lorentz

- Nobel Prize, 1902
- 1902 < 1905!

From Noble lecture 1902:

Permit me now to draw your attention to the ether. Since we learnt to consider this as the transmitter not only of optical but also of electromagnetic phenomena, the problem of its nature became more pressing than ever. Must we imagine the ether as an elastic medium of very low density, composed of atoms which are very small compared with ordinary ones? Is it perhaps an incompressible, frictionless fluid, which moves in accordance with the equations of hydrodynamics, and in which therefore there may be various turbulent motions? Or must we think of it as a kind of jelly, half liquid, half solid?

Clearly, we should be nearer the answers to these questions if it were possible to experiment on the ether in the same way as on liquid or gaseous matter. If we could enclose a certain quantity of this medium in a vessel and compress it by the action of a piston, or let it flow into another vessel, we should already have achieved a great deal. That would mean displacing the ether by means of a body set in motion. Unfortunately, all the experiments undertaken on these lines have been unsuccessful; the ether always slips through our fingers.

Having reached this point, we can consider the ether as a substance of a completely distinctive nature, completely different from all ponderable matter. With regard to its inner constitution, in the present state of our knowledge it is very difficult for us to give an adequate picture of it.

Lorentz transformation

$$t' = \gamma\left(t - \beta x\right)$$

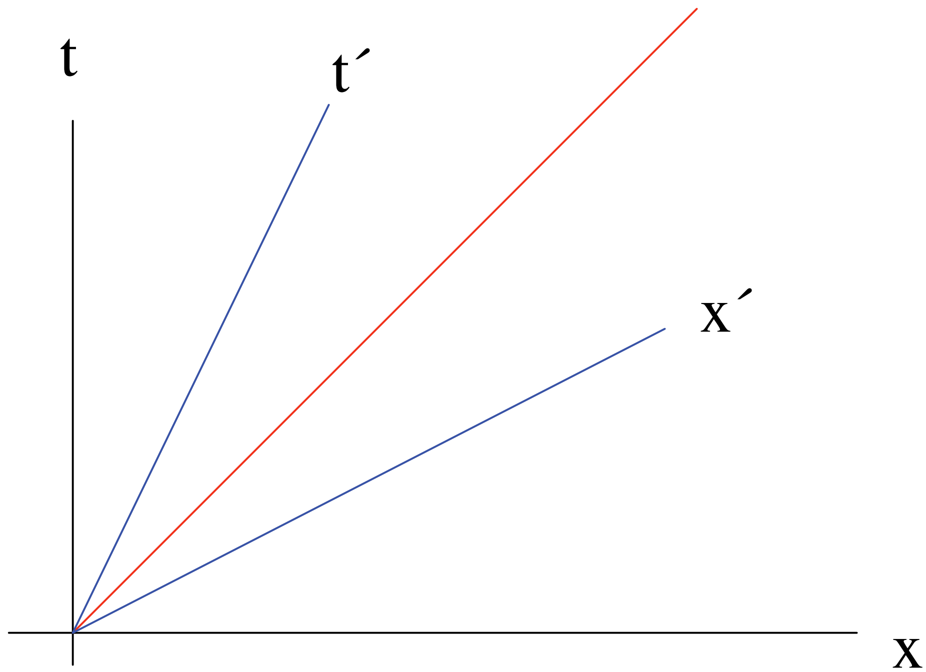
$$x' = \gamma\left(x - \beta t\right)$$

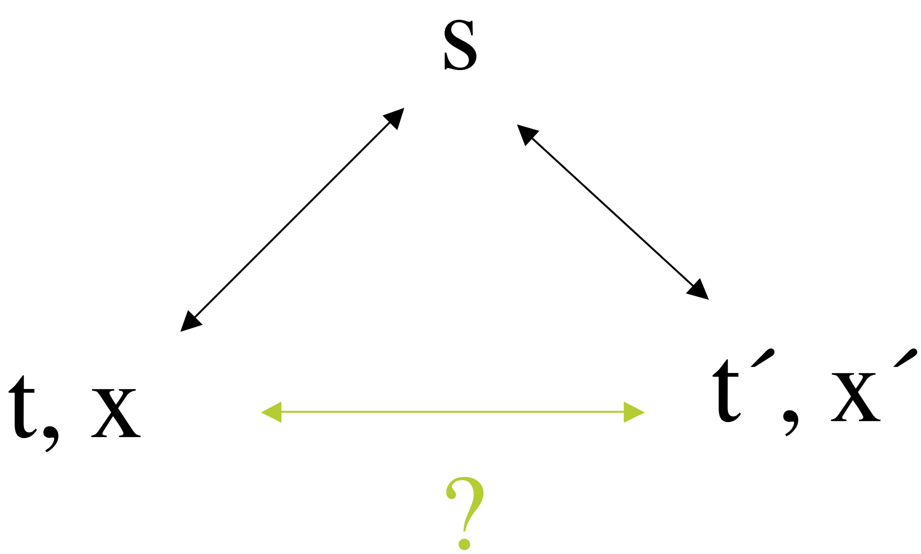
Review

- Spacetime interval
  - Invariant **by definition**
  - Definition?
    - Time read by inertial clock present at both events
  - Invariant speed of light implies

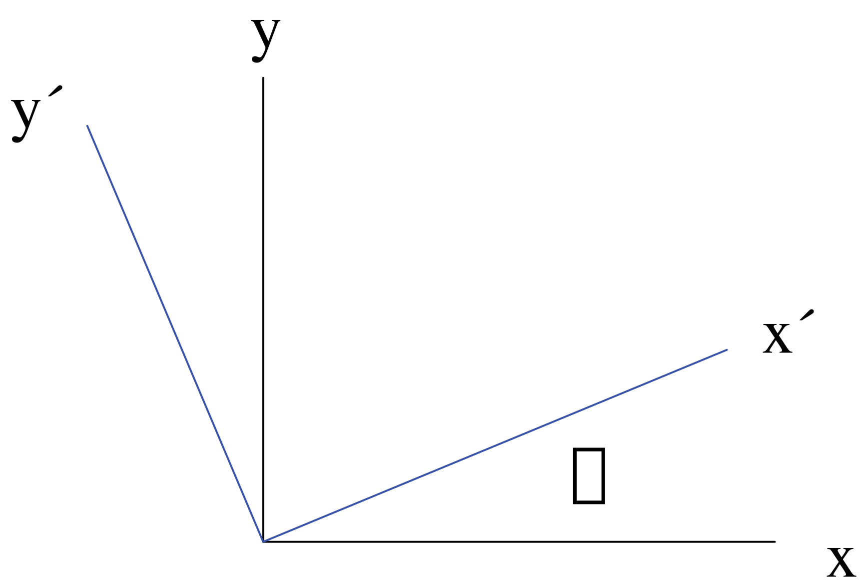
$$t' \neq t \qquad x' \neq x \qquad \text{but}$$

$$t'^2 - x'^2 = s^2 = t^2 - x^2$$


t
t´
x´
x


S
t, x
t´, x´
?

Euclidean space



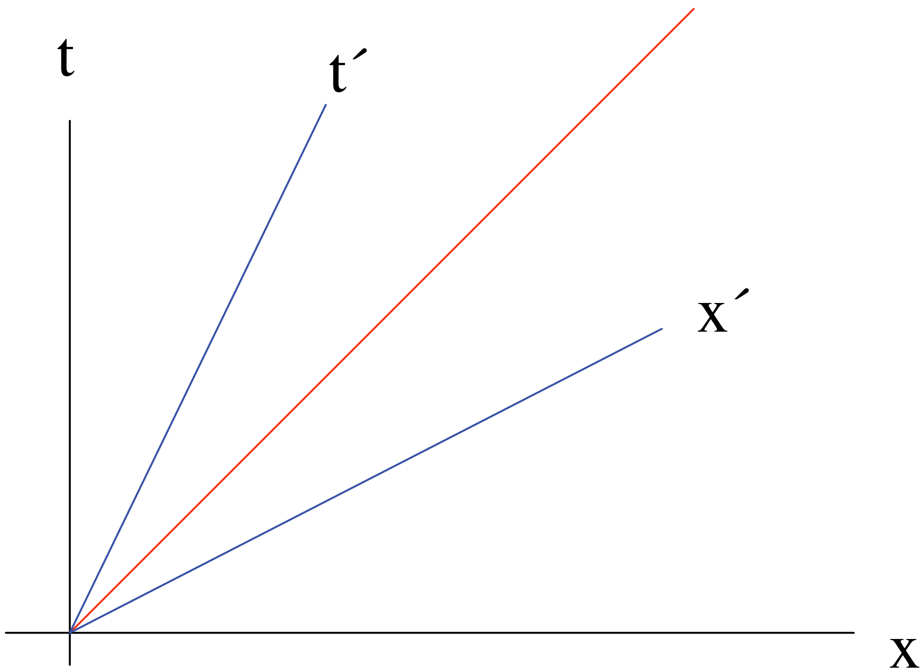
$$x'^2 + y'^2 = d^2 = x^2 + y^2 \quad \text{implies}$$

$$y' = \cos\theta\, y - \sin\theta\, x \qquad x' = \cos\theta\, x + \sin\theta\, y \quad \text{or}$$

$$y' = \gamma(y - \beta x) \qquad x' = \gamma(x + \beta y) \quad \text{with}$$

$$\beta = \tan\theta \qquad \gamma = \cos\theta = \frac{1}{\sqrt{1+\beta^2}}$$

Minkowski space



$$t'^2 - x'^2 = s^2 = t^2 - x^2 \quad \text{implies}$$

$$t' = \gamma(t - \beta x) \qquad x' = \gamma(x - \beta t) \quad \text{with}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Minkowski

$$t'^2 - x'^2 = s^2 = t^2 - x^2 \quad \text{implies}$$

$$t' = \gamma(t - \beta x) \quad x' = \gamma(x - \beta t) \quad \text{with}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

Euclidean

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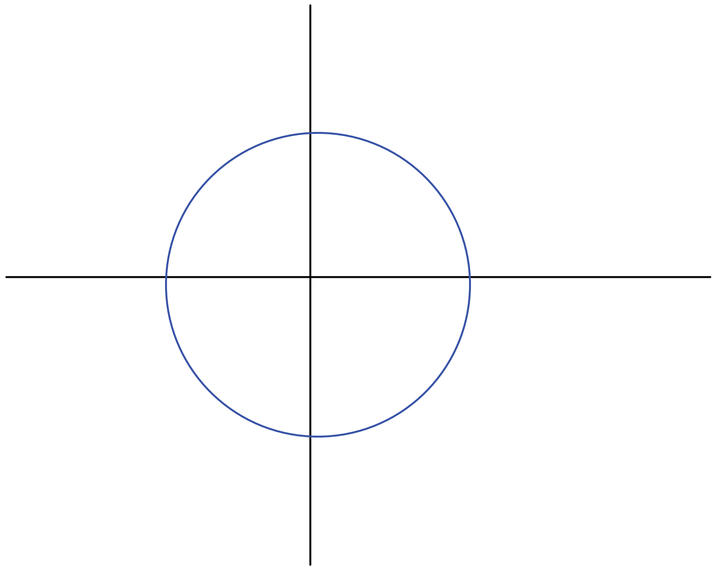
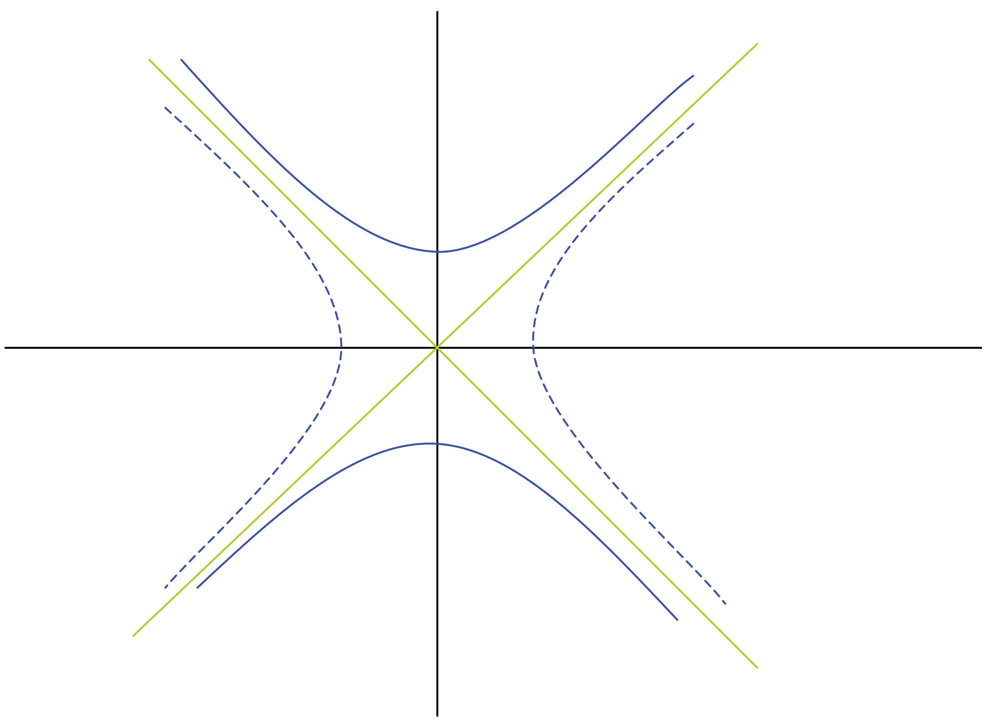
$$\beta = \tan\theta \quad \gamma = \cos\theta = \frac{1}{\sqrt{1 + \beta^2}}$$

Euclidean



Minkowski



Physics implies geometry

Euclidean geometry

$$x'^2 + y'^2 = d^2 = x^2 + y^2$$

$$y' = \cos\theta\, y - \sin\theta\, x \qquad x' = \cos\theta\, x + \sin\theta\, y$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right) \qquad \sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$$

Hyperbolic geometry

$$t'^2 - x'^2 = s^2 = t^2 - x^2$$

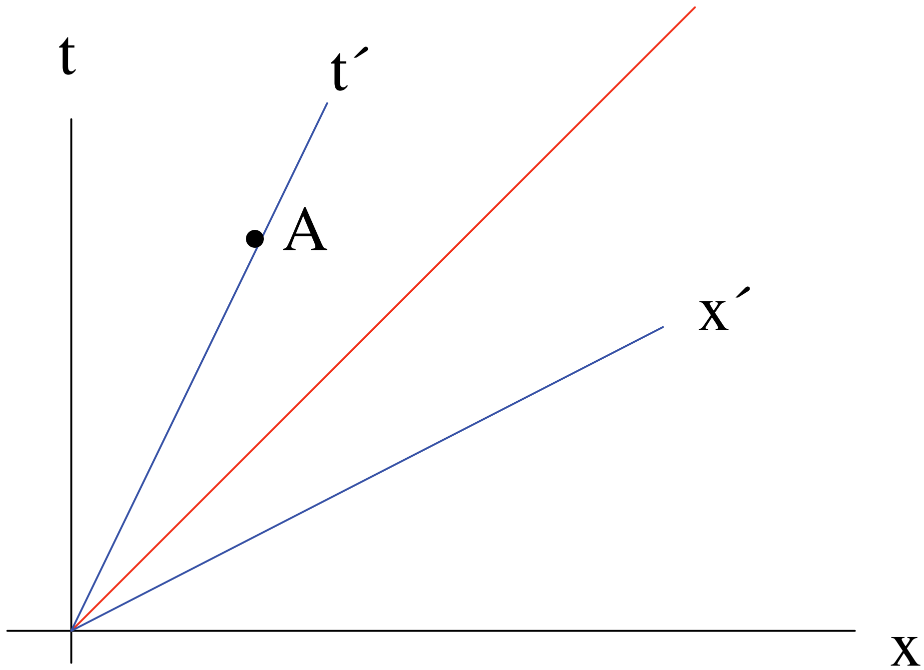
$$\cosh^2\phi - \sinh^2\phi = 1$$

$$\cosh\phi = \frac{1}{2}\left(e^{\phi} + e^{-\phi}\right) \qquad \sinh\phi = \frac{1}{2}\left(e^{\phi} - e^{-\phi}\right)$$

$$t' = \cosh\phi\, t - \sinh\phi\, x \qquad x' = \cosh\phi\, x - \sinh\phi\, t$$

$$\phi = \frac{1}{2}\ln\frac{1+\beta}{1-\beta}$$

Time dilation?

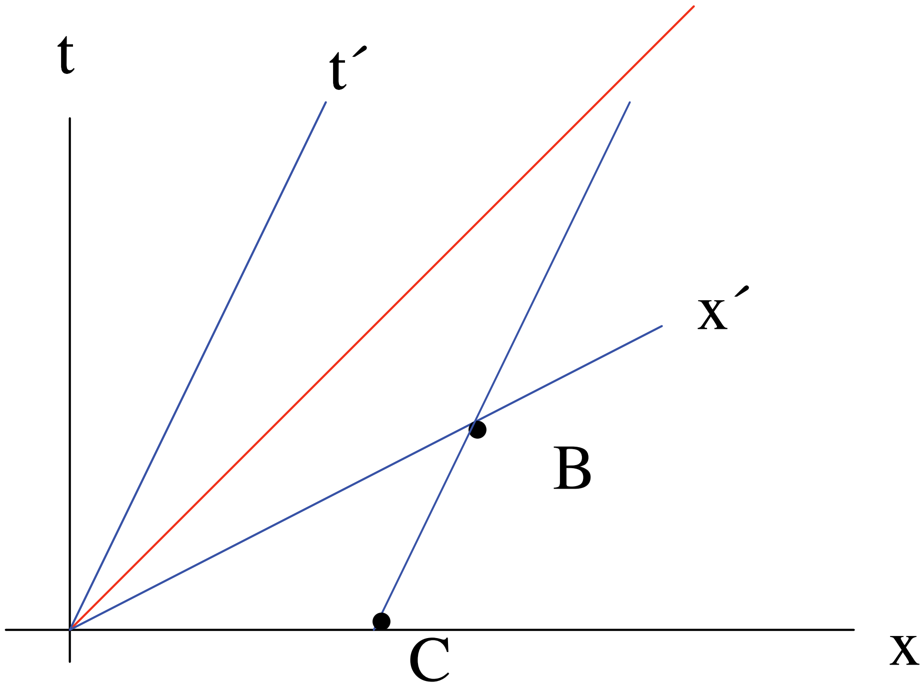


For event A: (t', x') $x' = 0$

$t = ?$ Use inverse Lorentz transformation.

$t = \gamma(t' + \beta x') = \gamma t'$ Good; we knew that.

Length contraction



For event C: t,x $t=0$ $x=L$ $x' = L_R$

$L = ?$ Use Lorentz transformation.

$x' = \gamma\left(x - vt\right) = \gamma x \Rightarrow L_R = \gamma L$

$L = \frac{1}{\gamma} L_R$