

Pion decay problem

The problem is to figure out the minimum velocity for pions so that they can travel a given distance without too many of them decaying away. The pions have a lifetime $T = 2.6 \times 10^{-8}$ sec., which is described in more detail in the previous note on unstable particles. I would like them to travel a distance $D = 3\text{m}$ in my rest frame and still have at least $1/e$ of them left to use as the beam for another experiment. At first one might think that they need to be going $v/c = D/(cT) = 0.385$ to make that work. In fact, one might also think that if the pions can go not faster than the speed of light, then the farthest they would ever go (on average) is $cT = 7.8\text{m}$. However, neither of these statements account for the fact that the proper time of the pion, by which its decay is controlled, is not the same as coordinate (or proper) time for me.

Since the pion is surely present at both its creation and its decay and since (by assumption) it travels with constant velocity between these events, its proper time is also the spacetime interval between those events. Thus to be left with at least $1/e$ of the pions created, I set $\Delta\tau = \Delta s = T$.

On the other hand, according to me in the lab, the spacetime interval is computed by measurements of the time and distance between the events via

$$\Delta s^2 = \Delta t^2 - D^2$$

and the time interval is $\Delta t = D/v$. The speed v of the pion that I want to know has finally appeared. So now it is just calculation. Substitute D/v for Δt and T for Δs and solve for v and then put in the numbers for D and T . That gives

$$v = 0.36$$

rather than the 0.385 mentioned above. So relativity helps me a little here. The pions do not have to be going quite as fast as I might have thought at first.

Also note that there is no upper limit to the D that I can get in a given pion proper time T for $D^2 = v^2 T^2 / (1 - v^2)$ which can be made as big as I want as $v \rightarrow 1$.