Units

Every specialty has its own convenient units. In high energy physics, we tend to measure many things in energy units. This is the same basic idea as we used in expressing length in time units. It seems odd at first, but after you get used to it, it saves a lot of work.

We have found it very convenient to measure length and time in the same units, e.g. seconds or nanoseconds. We did this by choosing the length units so that the speed of light comes out to be one, i.e. one length second is the distance that light travels in one second. If we need to get back to meters, we use the fact that when length is measured in meters, the speed of light is $c=3\times10^8$ m/s, so that (1 length-second)= 3×10^8 m.

$$x(\text{seconds})=x(\text{meters})/c(\text{meters/second})$$

With length and time measured in the same units, all velocities are dimensionless. They are dimensionless fractions of the velocity of light.

$$v(\text{dimensionless})=v(\text{meters/second})/c(\text{meters/second}).$$

Now for mass and energy: The units of energy are mass$\times$(velocity)^2. Since we have chosen units where velocity is dimensionless, mass and energy now have the same units. In high energy physics, where relativity is the most heavily used, it is more convenient to use energy units for both. The basic energy unit chosen is the electron volt eV. It is the energy an electron gets when it falls through a potential of one volt. In joules, it is $1.60\text{x}10^{-19}$ J. Actually, that is an inconveniently small energy, so usually millions of eV (MeV) or billions of eV (GeV) are used. In these units, the mass of an electron is 0.511 MeV, and the mass of a proton is 938 MeV. The conversion is

$$m(\text{MeV})=m(\text{kg})c(\text{meters/second})^2/(1.6\times10^{-13}\text{J}) \quad .$$

Since the units of momentum are mass$\times$velocity, momentum also has energy units now.

That is about all we can do just using the speed of light. We have mass, momentum, and energy in energy units, while length and time are in time units. However, there is another important constant of nature: Planck's constant. It is fundamental to quantum physics. We can use it to convert length and time to inverse energy units. Perhaps you have seen the famous relation (from de Broglie) $p=h/\lambda=\hbar k$. In this, p is the momentum, h is Planck's constant, $k=2\pi/\lambda$ is the wave number, and $\hbar=h/2\pi$. From this we see that the units of $\hbar$ are momentum $\times$ length or alternatively energy $\times$time. The value of Planck's constant is

$$\hbar = 1.05\times10^{-34}\,\mathrm{J\,s} = 6.58 \times 10^{-22}\ \mathrm{MeV\,s} \quad .$$

Now choose time units so that $\hbar$=1. That leaves length and time with units of inverse energy or equivalently inverse momentum.

$$x(\mathrm{MeV}^{-1}) = x(\mathrm{seconds}) / \hbar\ (\mathrm{MeV\ seconds}) \ .$$

To get back to standard units, you need to know two numbers $c=3\mathrm{x}10^8$ m/s and $\hbar c$=197MeV f. (The f stands for Fermi, one of the great physicists of the first half of this century. But 1 f is not how tall Fermi was; it's a femtometer: f=fermi=fm=10^{-15} m. The size of a proton is around 1f.) For example, for mass, 1 GeV=1 GeV/c^2=1.78x10^{-27}kg, and for length, 1/GeV=$\hbar$c/GeV=0.197x10^{-15} m. Simple, eh?