Unstable particles

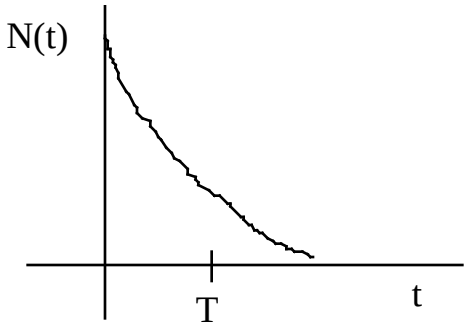
Unstable particles stick around for a while and then decay into other particles. For example, the positive pion π^+ can decay into a positive muon μ^+ and two neutrinos. If we start with a bunch of pions, they do not decay all at once. The process is probabilistic. In each unit of time, a certain average fraction of them decay. Let N(t) be the number of pions present at time t. Then

$$N(t+dt) = N(t) - dt \cdot N(t) / T$$

If T is large they decay slowly and if it is small they decay fast.
Now we can read off $dN = - dt \cdot N / T$, and integrate it to

$$N(t) = N(0)\, e^{-t / T}$$

(You don't have to take my word for it; you can check it by differentiation.) The constant T is called the lifetime of the particle. For the pion, $T = 2.6 \times 10^{-8}$ sec. The number of particles present at t = 0 is N(0).



After a time T, the average number of pions left has fallen to 1/e of it starting value, and in each following period T the number left falls by another factor of 1/e.

This is all in the rest frame of the pions. In the next installment, we need to see how this looks to another observer. That means that t , which might refer to any observer, should be replaced by τ , the proper time according to the pion.