Blackbody emission

As already shown, the energy density of blackbody radiation is

$$u = \frac{4}{c}\sigma T^4 .$$

I will show that the power radiated per unit area is

$$f = \sigma T^4 .$$

At equilibrium, the power radiated from a blackbody is equal to the power incident on a wall in the box containing the radiation. Thus, consider how much energy hits a wall in a time dt. The total volume available is cdtA, and the total energy in that volume is ucdtA. However, not all that energy can make it to the wall in a time dt. First of all, half of it is going away from the wall. Further, of the half that is heading toward the wall, only half will get there in time dt. Consider all the photons a distance z from the wall. To get to the wall in time, a photon must have $v_z dt > z$. The photons are equally likely to be going in any direction. Thus they are distributed according to the angular surface element $d\varphi d\theta \sin\theta = d\varphi d\cos\theta$. This says that all values of $\cos\theta$ are equally likely. But $v_z = c\cos\theta$ so all values of v_z are equally likely. As already mentioned, the half with $v_z < 0$ are immediately eliminated. When z=0, all the photons with positive v_z can get to the wall in time. When z=cdt, none of them can. The average over the volume out to cdt from the wall is 1/2. Thus 1/4 of the photons in the volume cdtA get to the wall. That gives for the flux (power per unit area onto the wall)

$$f = \frac{ucdtA}{4dtA} = \sigma T^4 .$$