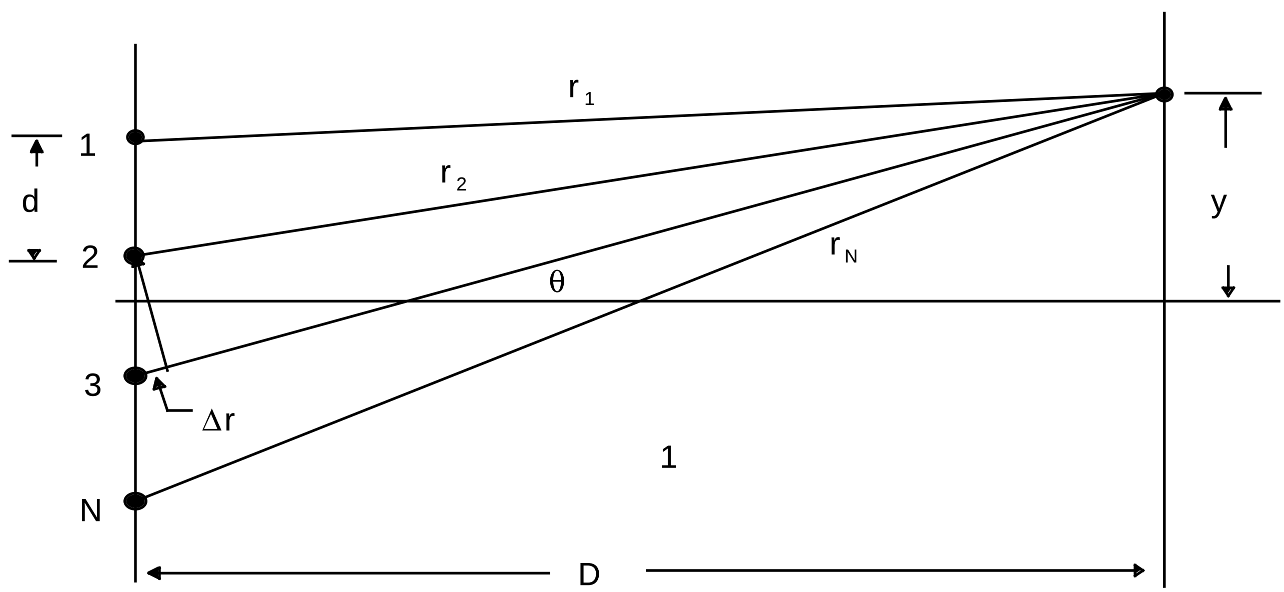
Multiple source interference and single slit diffraction

We have done the case of two-source interference including the intensity patttern. In that case, it was assumed that each source was a point (really just smaller that a wavelength is good enough). To do diffraction from a single coherent source of finite width or from a finite width slit, it is easiest to first do the multiple source interference problem. This case has is own interest too, since it is applicable to diffraction gratings, phased array antennas, and other practicle things. The calculations get a little harder but the good news is that there are no new concepts; it is an exercise in applying things about interference that have already appeared in two-source interference.

Multiple source interference

It's all in the picture. Once we have the picture and the notation, its a straightforward calculation.



As before, $\Delta r = d \sin \theta$ and $\varphi = k\,\Delta r = k\, d \sin \theta$, and for simplicity $y/D \ll 1$, $\theta \ll 1$, $Nd \ll D$.
To get the wave at the observation point, we must add the waves from each of the N sources. Again we do the simplest case where the amplitues are the same and the sources are in phase.

$$f = f_1 + f_2 + f_3 + \ldots + f_N$$

$$f_1 = A\cos(kr_1 - \omega t) \qquad f_2 = A\cos(kr_2 - \omega t) = A\cos(kr_1 - \omega t + \varphi)$$

In general for the n-th term, $f_n = A \cos(kr_1 - \omega t + n\varphi)$ so that

$$f = A \Sigma_0^{N-1} \cos(kr_1 - \omega t + n\varphi).$$

Then an identity proven in the companion document *sum formula* gives

$$f = A \cos(kr_1 - \omega t + \{N-1\}\varphi/2) \sin(N\varphi/2)/\sin(\varphi/2) .$$

Now we just apply the same reasoning that we used in the two-source case to get the average intensity

$$I(\theta) = \{I(0)/N^2\} [\sin(N\varphi/2)/\sin(\varphi/2)]^2 .$$

This is the intensity pattern for N sources. You should try to figure out what this looks like. We will spend some time in class understanding this result.

Diffraction

The next problem to do is diffraction from a source or slit with a finite width a. At first, that seems very hard. But with the work we have already done for N-source interference, it is easy. The idea is to divide the finite source into N sources of strength A/N and separation a/N, use the above result and then let $N \to \infty$.
Introduce $\beta \equiv N \varphi = N k d \sin \theta = N k (a/N) \sin \theta = k a \sin \theta$. Then it is simple; just put $\varphi = \beta/N$ in the N-source result to get

$$I(\theta) = \{I(0)/N^2\} [\sin(N\varphi/2)/\sin(\varphi/2)]^2 = \{I(0)/N^2\} [\sin(\beta/2)/\sin(\beta/2N)]^2$$

$$= \{I(0)/N^2\} [\sin(\beta/2)/(\beta/2N)]^2$$

$$I(\theta) = I(0) [\sin(\beta/2)/(\beta/2)]^2 \qquad \text{with} \qquad \beta = k a \sin \theta.$$

This is the result for the intensity for diffraction. Again, you should spend some time with this formula to figure out what it says and to verify that it gives the difraction "nulls" where the text claims that they are.