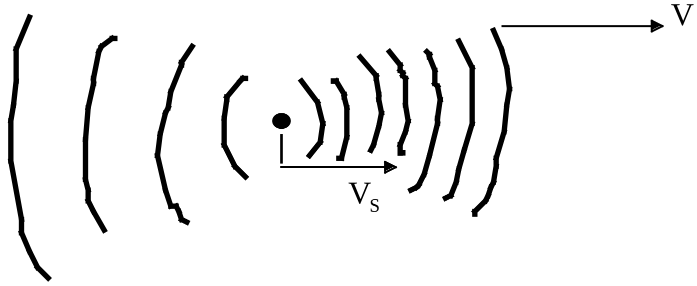
Doppler shift

The Doppler shift is an observed change of frequency of a wave due to the relative motion of the medium, the observer, and the source of the wave. We have described how this works for light where there is no medium, and it is only the relative motion of the source and observer that counts. Let's discuss it for a mechanical wave like sound with a physical medium. You may have noticed that the pitch of a siren sounds higher as approaches and lower as it recedes.

Sound is supported by a mechanical medium (the air), which provides a preferred frame of reference. Thus I will measure all velocities relative to the air. In the case that the wind is blowing, the velocity of the air relative to the ground must be accounted for. I will restrict myself to one dimensional problems. The x-axis is at rest relative to the air. All motion of the source and observer is along the x-axis. Velocities are relative to the x-axis and can have either sign. Suppose that a source of sound with natural angular frequency ω is moving with velocity V_S along the x-axis. I will discuss the case of motion in the positive direction, but the formulas are correct for all signs.

The sound emitted by the source moves at a fixed speed v of about 300m/s relative to the air. If the wave is going to the right, its velocity is V=+v, and if it is going to the left, the velocity is V=-v. The basic point is that the source is moving relative to the waves that it emits. The wave crests moving to the right get compressed, and those moving to the left get separated.



The period of the source is $T=2\pi/\omega$. In a time t, the source emits t/T crests. The first crest has traveled a distance Vt to the right. The source has traveled a distance $V_S t$ to the right by the time it emits the last crest. The separation between the first and last crest is $d=(V-V_S)t$. Relative to the medium t/T crests go by in a time $t'=d/V$. This gives

$$\omega' = \frac{2\pi}{T'} = 2\pi\frac{t/T}{d/V} = \frac{2\pi}{T}\frac{Vt}{d} = \omega\frac{V}{V-V_S}\ .$$

This says that $\omega' > \omega$ when the source and the wave are in the same direction (as it should).

Now consider an observer over to the right of the source and moving with a velocity V_O. In a time t'', the wave moves a distance $d'=(V-V_O)t''$ relative to the observer. In that distance, there are d'/λ' crests. ($\lambda'=VT'=2\pi V/\omega'$). Thus,

$$\omega'' = 2\pi\frac{d'/\lambda'}{t''} = \frac{2\pi}{\lambda'}\frac{d'}{t''} = \omega'\frac{V-V_O}{V}\ .$$

In the last equation, ω' is the angular frequency heard by an observer at rest relative to the air. For V and V_O positive, $\omega''<\omega'$ as it should be. If the source is moving then, ω' is not ω. Replacing ω' by its expression above gives

$$\omega'' = \omega \frac{V - V_O}{V - V_S}$$

. In all these results, ω could just as well be replaced by the ordinary frequency f since it is just a factor of 2π that relates them, and it will appear on both sides of the equations.

Consider an example that has everything. The ol' north wind is blowing at 10m/s. I'm riding my bike north at 5m/s. A car, going 30m/s, is approaching me from the north, and honks. The true horn frequency is 1000Hz. The speed of sound is 300m/s. What frequency do I hear? It is very easy to make sign errors in these problems, so I must be very careful. I choose my x-axis pointing north and at rest relative to the air. First I must give all the velocities in this frame. For me, V_O=5m/s+10m/s=15m/s. For the car, V_S=-30m/s+10m/s=-20m/s. For the sound, V=-300m/s. Thus I hear the frequency

$$f'' = 1000\,Hz \frac{-300 - 15}{-300 - (-20)} = 1125\,Hz.$$

The relations above for the Doppler shift are correct for sound, which has an associated material medium, but are not correct for light, which does not. For light, the result we got before can be rearranged to conform with the present notation to give

$$\omega'' = \omega \frac{|V - w|}{\sqrt{V^2 - w^2}} \quad \text{with} \quad w = V_O - V_S.$$

It is the relative velocity w of the observer and the source that appears. This hints at frame independence. But there is still V. Before V=±v was the velocity of the wave relative to the medium. The strange thing about light is that its speed is $c=3\times10^8$m/s in any frame! Thus, there is no reference to the medium, and the result is true in any frame. This is the essential point in Special Relativity. There is no medium with respect to which velocities should be referred. There is no material stuff which supports light waves. You can amuse yourself by showing that when w is small relative to c (as is the case in ordinary human experience), then the forms of the relations for light and sound are the same.

Many of the most interesting uses of the Doppler shift are for light rather than waves in material media. If observer and source are approaching, the light is shifted to higher frequency and shorter wavelength. If they are separating, the shift is to lower frequency. In the first case, light in the middle of the spectrum is shifted to the blue while in the latter case it is shifted to the red. These are called blue shifts and red shifts, respectively. In fact, any Doppler shift to a higher frequency is often called a blue shift and one to lower frequency a red shift even if the EM radiation is elsewhere in the EM spectrum. Doppler shifts are the only way that we have to measure the velocity of very distant astronomical objects. The expansion of the universe, which led to the big bang theory, was discovered in this way. The farther a galaxy is from us, the faster it is receding from us. This is called the Hubble expansion.