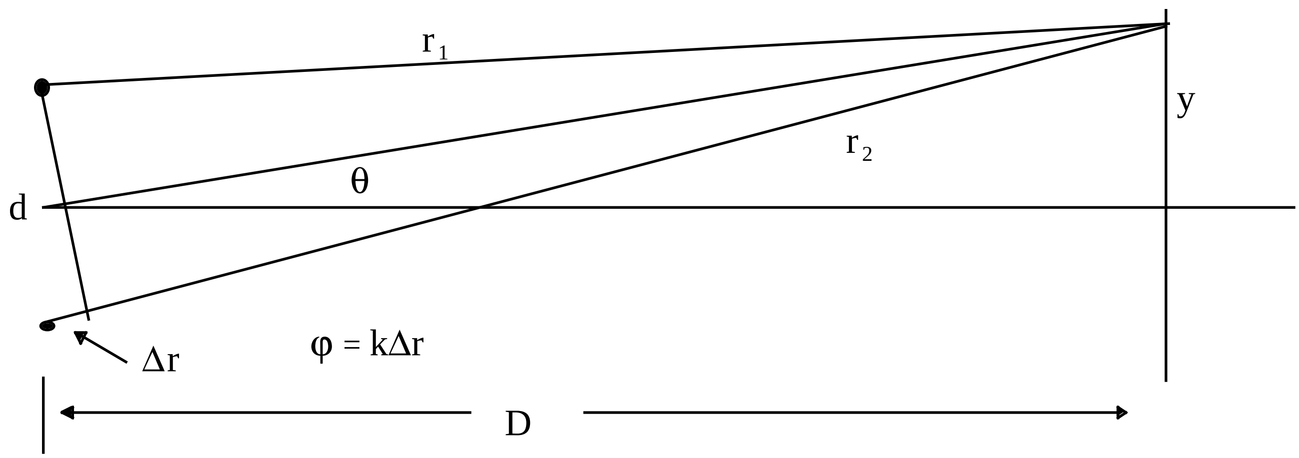
Two-source intensity pattern

This gives the derivation of the two-source interference intensity pattern. For simplicity, the two sources are taken to have the same phase and amplitude. They are separated by a distance d and are being viewed at a large distance D. I will assume that D » d, that $\theta < 1$, and that d » λ in some places. These assumptions are just simplifications, they are not essential to the basic physics result. What *is* essential is that the sources be coherent *i.e.* that they have the same wavelength and a fixed phase relation.



$\Delta r = d \sin\theta$ $\qquad\qquad$ $\varphi = k\, d \sin\theta$

$$f_1 = A\cos(kr_1 - \omega t) \qquad\qquad f_2 = A\cos(kr_2 - \omega t)$$

At the observation point, waves from both sources arrive and add (superposition)

$$f = f_1 + f_2 = A\,[\,\cos(kr_1 - \omega t) + \cos(kr_2 - \omega t)\,]$$
$$= 2\,A\cos(k\{r_1+r_2\}/2 - \omega t)\cos(k\{r_2-r_1\}/2)$$
$$= 2\,A\cos(k\{r_1+r_2\}/2 - \omega t)\cos(\varphi/2)\ .$$

(The step from the first to the second line is just a trig identity.) In this kind of problem, it is the intensity of the wave that is usually considered. You will recall from our discussion of energy, power, and intensity that the intensity is proportional to the square of the amplitude of the wave. Thus consider

$$f^2 = 4\,A^2\cos^2(k\{r_1+r_2\}/2 - \omega t)\cos^2(\varphi/2)$$

This is the instantaneous intensity. The first $\cos^2$ factor has ωt in its argument and is therefore a function that oscillates very rapidly between zero and one. In many cases, these rapid oscillations are either not of interest or are (as in the case of human eyes and light) far too rapid to be detected. Then it is appropriate, to consider the time average intensity, which will be called I. Since the average of $\cos^2$ is 1/2, the intensity as a function of the observation angle is

$$I(\theta) = 2 A^2 \cos^2(\varphi/2) \qquad \text{with} \qquad \varphi = k\, d \sin\theta$$

In the forward direction, $\theta = 0$, and $I(0) = 2A^2$. You should be able to show for yourself that this is *four* (not two) times larger than the $\theta = 0$ intensity would be with just one of the sources. Why is it four and not two? If it is only relative intensities that are of interest, then it is easier to write the average intensity as

$$I(\theta) = I(0) \cos^2(\varphi/2) \quad .$$

To develop your understanding or this intensity formula, you should be sure that you can make and interpret a graph of I vs. θ, identify the maxima and minima, and show that the positions of the maxima agree with the result in the text. It is sufficient to do this in the approximation of small λ/d and small θ. You might also look at the average of this intensity over viewing angle θ and show that that *is* two times the single source intensity. Can you explain that?