# Problems for problem set 6

For questions 1 and 2: Consider a potential step 10eV high so that V(x)=0 for for x<0 and V(x) = 10eV for x>0. You might want to look at the document Step potential summary, which summarizes the class discussion and also has a brief comment of fluxes (needed in 1b below).

1. Electrons with a kinetic energy of 15eV are beamed at the step from the left.
   a) Use the Schroedinger equation to get the momentum of the electrons for x>0. How does your result compare with what you would get from classical reasoning?
   b) What fraction of the electrons travel to right past the step? In doing this, think in terms of quanton fluxes.

2. Electrons with a kinetic energy of 5eV are beamed at the step from the left. Roughly what is the distance to the right of x=0 that there is a reasonably large probability to observe an electron?

3. Hydrogen atom estimate.
Consider the hydrogen atom. Classical physics cannot account for the size of the atom or the energy that it takes to remove the electron. With the Schroedinger equation in its 3-dimensional version, one can do a detailed calculation that gives the size and ionization energy. However, the calculation is a bit involved, and it is easy to lose sight of what is really going on. Here is a "back-of-the-envelop" calculation that can only be expected to be roughly correct (to within factors of 2 and $\pi$ and such) that makes it easier to see what is really happening. However, apparently by accident, it actually gives the correct answer with all the numerical factors right!

Work through the following steps explicitly:
Assume that $\psi_E(x)$ for the electron is large in a volume around the proton with radius R and is very small outside that volume. Then the kinetic energy term in the Schroedinger equation is something like

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_E}{\partial x^2}+\frac{\partial^2\psi_E}{\partial y^2}+\frac{\partial^2\psi_E}{\partial z^2}\right)\rightarrow\frac{\hbar^2}{2m}\frac{1}{R^2}\psi_E \ .$$

(This is a given; don't try to derive it. But you should be able to see why it is a plausible estimate.) Put this into the Schroedinger equation with the potential as given in the text discussion of the Bohr model

$$V(x,y,z)\rightarrow-\frac{ke^2}{R} \ .$$

Cancel out the common factor of $\psi_E$ to get an expression for E. Minimize this with respect to R. Find the resulting expression for R and give its value. (You will need the value of $ke^2$ in the text.) Put this back into the previous expression for E to get a new expression for E with R eliminated. Give its numerical value. Compare with the results of the Bohr model.

Although this little calculation does not yield as much as the Bohr model, it is a more honest approach in that it is closer to what happens when the full equation is solved.