Uncertainty principle introduction

$e^{ipx/\hbar}$

In class, I argued that the probability amplitudes for quantons (see Moore, glossary, page 287) of definite wave number need a factor of

$$e^{ipx/\hbar} = e^{ikx} \quad \text{with } p = \hbar k.$$

$\langle bra|ket\rangle$ *notation*

There are several notations for probability amplitudes. One is the bracket notation. The amplitude that a system prepared with initial properties I is then observed with final properties F is written < F | I >. As a probability amplitude, this is a complex number. A general property of amplitudes is that opposite process with the system prepared in state F and detected in state I is $< I | F > = < F | I >^*$, the complex conjugate of < F | I >.

Thus the amplitude that a quanton prepared with momentum p is observed at position x has a factor

$$\langle x|p\rangle \propto e^{ipx/\hbar} \text{ and } \langle p|x\rangle \propto e^{-ipx/\hbar}.$$

Uncertainty relation

Lets say that we have prepared a quanton so that we know it is most likely in a region of width Δx near the origin. Thus the amplitude ψ(x) to find it at position x looks like
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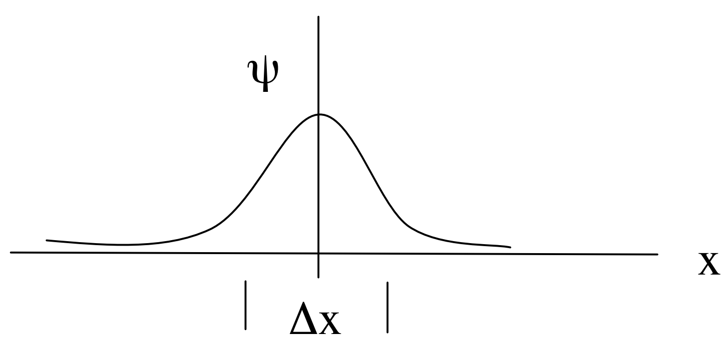

What is the amplitude < p | ψ > to detect this quanton with momentum p? We do not know this directly, but we can think of it like a quanton going through holes in a screen. The quanton can go from state ψ to position x and then to momentum p, and we sum over all the "holes" x.

$$\langle p|\psi\rangle = \int dx \langle p|x\rangle\langle x|\psi\rangle = \int dx \langle p|x\rangle \psi(x) = \int dx e^{-ipx/\hbar}\psi(x)$$

The probability for find it with momentum p is $P(p) = | < p | \psi > |^2$.

Suppose that $p/\hbar << 1/\Delta x$. Then in the region of x where ψ is large, the phase $px/\hbar$ is near zero, and the phase factor $e^{-ipx/\hbar}$ is near one. The integral is relatively large, $< p \mid \psi >$ is relatively large, and the probability to find the quanton with momentum p is relatively large. Suppose, on the other hand, that $p/\hbar >> 1/\Delta x$. Then both the real and imaginary parts of the phase factor oscillate between plus and minus one very rapidly. The positive regions tend to cancel the negative regions in the integral. The probability to find the quantun with $p/\hbar >> 1/\Delta x$ is small. Thus if we know that the quanton is likely to be in a region of width Δx, then it has a momentum spread of width Δp at least $\hbar/\Delta x$. I.e. $\Delta p \Delta x \sim \hbar$.